

PHILOSOPHY OF FINANCE

Definitive Smile Model: Part II

Black-Scholes is right and significant only to the extent that it is not true. This issue we look at what arises from a discussion of true versus right.

The so-called *nesting of models* seems to be the most recent fashionable exercise with respect to the truth project in quantitative analysis. For instance Bakshi and Cao (2003) argue in a recent empirical study that a double-jump option pricing model taken from Duffie, Pan and Singleton (2000), which improves on the previous model (Bates 1996, which in turn improved on the model before (Heston 1993) in adding underlying jumps to stochastic volatility) in offering the possibility of adding volatility jumps correlated to the underlying jumps, performs better both in matching the in-sample vanilla options and in pricing the out-of-sample options. Not forgetting Lipton, who argues that the ‘universal volatility model’ which improves on all of the local volatility, jump-diffusion and stochastic volatility models in mixing all their characteristics, performs better in terms of pricing the exotic options. The impression one gets from this argumentative zeal is one of a converging sequence of models, bound to reach the final nest where truth must be lying. I wonder how the exotics would fare in the double-jump model, and whether a bit of its

vanilla explanatory power should not be sacrificed in order to account for the barriers. At any rate, turning one’s attention to the exotics would imply a break in the thrust of the argument of Bakshi and Cao and in the push to the truth about the vanillas. The same break occurs in Lipton, if one starts worrying about the possibility of a change in the price structure of the barriers for a given vanilla price structure.

The right alternative to a false Black-Scholes model, I think, is not to look for the true substitute but to drop the whole metaphysical notion of *truth in an option pricing model*. Although Black-Scholes is clearly false in the sense of not corresponding to empirical fact about option prices, I want to argue that it is *valid*, in a new enlarged sense of validity. If the Black-Scholes model is still being used by traders and practitioners all round, then it has got to be valid, and this validity has got to be independent of the true-false dichotomy. Last issue I stressed that the two important things in Black-Scholes are the notion of dynamic hedging and the synthesizing of option prices in the implied volatility number. What the first really did is allow the traders to link option value to a concrete rule of action. The necessity to update the option *delta* with the Black-Scholes formula and to rebalance the hedge every time the underlying moved was the real reason why the Black-Scholes model was used in effect. Given the freedom that the option trader enjoyed in setting up the implied volatility number, it should have been suspicious from the start that the Black-Scholes formula might have had another motivation. Option valuation was made *effective* through the concrete link that the delta provided with the underlying. And valuing options effectively was no longer a matter of applying the pricing formula punctually

and theoretically, but required from the trader that he consistently monitored and *followed* his option trade. Surely enough he could lock the option value by neutralizing his delta exposure, but this very move suggested that he should get back to his option trade every now and then, and gave meaning to this constant revisiting. Following the rule of delta hedging and delta rebalancing inscribed the option value in a chain of coordinated actions instead of leaving it as a theoretical result on the trader’s spreadsheet. It turned the option into a *relational* concept which now involved the whole functional relationship with the underlying and no longer stood alone in abstraction¹.

As for the second important thing – the expression of option prices in terms of implied volatility numbers – it provided the option traders with a new and very efficient language. Traders were able to relate to the (implied) volatility they were buying, selling, or trading off, more easily than they did to the naked option prices, and the Black-Scholes model which inspired all this with its flat volatility assumption, never was an impediment to the actual multiplication of implied volatility numbers across the option chains, and to the capacity of the language to adapt itself to situations pretty much at variance with the original Black-Scholes world.

The philosophical point I am trying to make, which will help banishing truth altogether as an irrelevant category in our case, is that the Black-Scholes model has bestowed *meaning* on options and on option trading through the *algorithm of delta hedging* and the *language of implied volatility*, and that meaning is not of a such nature as to fall under the scrutiny of metaphysical truth or to be deemed true or false. The realm of mean-



Delta hedging is the right thing to do – and the trader doing it shows a perfect understanding of the meaning of options, even though he may not know the truth

ing, also known as the realm of validity, is philosophically distinct from the realm of truth. And I claim that Black-Scholes is valid because meaning is a much richer category than truth. Think that we can use language meaningfully, and for that matter compose poems and create metaphors, or propose scientific theories and advance wild interpretations of the physical world, without necessarily speaking truthfully.

True, the trader may be flying in the face of the theoretical Black-Scholes model when he updates both the underlying price *and* the implied volatility number in his formula and rebalances his hedge accordingly, still it cannot be claimed that he therefore represents a falsity. On the contrary, delta hedging is the right thing to do – this is the main lesson from Black-Scholes –, and the trader doing it shows a perfect understanding of the meaning of options, even though he may not know the truth about them. Now Black-Scholes may be the *wrong* model to use for delta hedging in the presence of smiles (given jumps and stochastic volatility), and we may be willing to start looking for the *right* model. The fact remains that the valid dichotomy here is the right-wrong dichotomy, not the true-false. Right and wrong do not partition the space of reasons the same way that true and



The unsayable is not true, but there is something it is right about

false partition the space of facts. You may be doing or thinking the right thing for the right kind of reason, without there necessarily being a fixed reference against which you can justify your action or your thinking.

When Lipton and Hagan argue that their model is the right model because it gives the right barrier option prices or produces the right option hedges, their argument is a truth claim in disguise, not a validity claim. Hence our criticism. Indeed the barrier option prices and the vanilla option hedges are the fixed reference they relate to and the ultimate truth-maker they seek. By contrast, what would be a valid and much richer model (valid in our extended sense of validity, richer in the sense that validity and meaning precisely exceed truth) – in a word, what we would call the *right* model – is a model where you would explicitly include the barrier option prices and the vanilla option deltas in the calibration. And we say that the model is right (and not just true) because it depends on no external, “fixed” reference which may very well vary the next day, but incorporates the *variability* of the reference itself. It turns the concept of the *right smile model* into a relational and relative concept: the model will give the *right* barrier option prices, or the right delta hedges, simply because it relies on a law of logic (even a syllogism) not on a matter of fact; it will give the right price for the barrier options when it is calibrated to the right barrier options, and it will produce the right delta hedges when it is calibrated to produce them.

The significance of Black-Scholes

To really assess the significance of the Black-Scholes model and what it meant to both the science and the history of the science, and to fully

appreciate what it takes to really think about Black-Scholes, think what our thinking would look like if Black-Scholes were true. If hedging were continuous and if we lived in a world of underlying Brownian motion with constant (non stochastic) volatility, options would be redundant. They wouldn’t exist except by name. All that would remain to do is to buy or sell the underlying (and you would definitely find somebody prepared to take the opposite bet, in this perfectly random world), or to invest an initial fee in a certain combination of the underlying and the riskless bond, to be able to run a self-financing dynamic trading strategy which may result, for instance, in being long the underlying at a certain level, at a certain date, if it trades above that level at that date, or in being short it at a certain level, if it trades below that level. Conversely, you may sell that combination for a certain fee, and run the opposite self-financing dynamic trading strategy in order to preserve that fee, no matter the outcome of the underlying at maturity. Options would exist only by name, and the underlying would be the only thing worth buying or selling or trading in ever more sophisticated strategies. And should it turn out that options must exist, by some metaphysical decree, beyond the mere naming of those self-financing dynamic strategies, why would anyone buy them or sell them? Wouldn’t everybody agree on their initial value and their outcome? Since you can personally perfectly replicate any contingent payoff, all you would need is a party to your trades in the underlying. No option market *per se* would come to exist.

What we are really saying is that if Black-Scholes were true, what Black-Scholes would really have to say (“Options exist and they can be traded. You can buy them, sell them and even hedge them, etc.”) would not be true or false, or right or wrong. It would really be unsayable. Black-Scholes would really have nothing to say.

Black-Scholes is right insofar as it is not true

Fortunately, Black-Scholes is not true, and this is why we have something instead of nothing. As Alberto Coffa would say, “*the unsayable is not true, but there is something it is right about.*” And what Black-Scholes is right about is precisely this, in Black-Scholes, which looks outside the closed formula and outside the complete market paradigm and its tautological consequence for options. Black-Scholes is precisely right in having bestowed on options and option markets the meaning that we have been talking about. And what is so amazing about the Black-Scholes model, and definitely distinguishes it, and the history of the science that will follow from it, from any other history of science, is the extraordinary philosophical pressure that is exerted on it the minute it is subjected to reflection. Never before has a model or a theory or a framework been so finished and so closed on paper and so eager to crack open under philosophical questioning. *Black-Scholes is right insofar as it is not true.* Anything meaningful, and historical, and thought-provoking that Black-Scholes may have to say, has nothing to do with Black-Scholes and everything to do with smiles. Options exist (independently of their hedging strategies of course: otherwise how could we even start talking of hedging *them*?) only insofar as the hedge is not perfect and there is leeway in the choice of the hedging strategy. And option markets exist only insofar as the language of implied volatility has got more than one word.

The process of objectification and the true science in Black-Scholes

Now we can see why the two most significant strands in Black-Scholes, the dynamic hedging story and the implied volatility story, are the true things worth generalizing and reflecting upon. Once the philosophical picture is set in the right frame, and the Black-Scholes model is no longer followed for the something *true* but for the something *right* that it has to say, we understand where all the robustness comes from. Black-Scholes seems so inseparable from options and option talk because it was first to insert the option value into the algorithm of delta hedging and the language of implied volatility. It thereby granted options a special kind of being: a ‘being

objective’ which is at once more significant than ‘being a name’ and far more robust (far less risky and unstable) than ‘being true.’ The Black-Scholes model turned options into scientific and linguistic *objects*. The original theory may be simplistic and we may have abused of the original single-worded language, the fact remains that the delta hedging algorithm has contributed to the process of “objectification” of the option (as a neo-Kantian would say), or in other words to “the construction of its being *as object* through conceptual determination.” As for the implied volatility language, it has provided an effective translation of option prices and option markets. When traders relate to the Black-Scholes model, they do not really care whether the model, as model, is true, and whether it relates to some transcendent reality. All they care about are the *objects* and the functional relations between them². They care about the option and the delta as inter-operative concepts. What I am trying to say is that the scientific moment that one should try to capture in Black-Scholes is the moment of the *sending* of the strands (Brownian motion as the simplest way of breathing life and time value in the option, implied volatility as the single knob to calibrate the model with, and dynamic replication as the operative rule), not the moment when the strands meshed with each other in a single fateful knot, and gave us the closed-form formula and the complete market, thus ending philosophical thinking before it even started.

The science that we would like to capture and nurture in Black-Scholes is not the bit that argues from Brownian motion to the continuous perfect replication to the Black-Scholes PDE to the analytical formula. For this is only a clever mixture of stochastic calculus and no-arbitrage principle, which takes advantage of the continuous-path property of Brownian motion and the ability of a continuously rebalanced self-financing portfolio to be immune against the Brownian innovation. This “pencil-and-paper” Black-Scholes does not really interest us. What science we see in Black-Scholes is the part that gave birth to the history of the science. It is the part concerned with the objectification that we talked about earlier. As our neo-Kantian philosopher of

If the history of the science were to be rewritten, Black and Scholes would really have to keep their paper hidden from the eyes of the public

science would go on to say: “The *fact of science* is the fact of objectification at its most developed stage, and philosophy’s task is to grasp the categories of objectification governing scientific development.”³ The part in Black-Scholes corresponding to the “fact of science” is no doubt the part that makes options objective, not the one that makes them redundant – the part that initiates philosophical thinking, not the one that evacuates it. It is the part which literally occurs outside the closed-form formula and speaks distinctively of options, of option hedges, and option implied volatility smile. It wouldn’t cross the mind of the first option trader anyhow that options may be redundant, and that they may not have their own market, quite independently of the underlying.

The history of the science

Now think that the original motivation of Black-Scholes and Merton was to provide the traders with tools to rationally price and possibly arbitrage those options! Surely enough, the assumption of lognormal distribution of asset returns must have seemed to them the most attractive initial step to get the problem going. And how surprised Black and Scholes must have been to find, as a result of this single step, that options and option markets were being dismissed completely! If the history of the science were to be rewritten, Black and Scholes would really have to keep their paper hidden from the eyes of the public. Any option pricing and hedging model would have been good for publishing, except the original Black-Scholes! This is why we’ve been urging that, although the Black-Scholes model is undoubtedly a historic finding and although the Black-Scholes language still permeates the totality of our conceptual dealings with options – even the word

“smiles” implicitly refers to Black-Scholes –, we should really think of options as if Black-Scholes had never existed. This means we should not try to save the complete market paradigm at all costs, or look preferentially for models which result in analytical pricing formulae. All these things, all these worries and the research programs that they spawned, should really disappear from our sight when we interpreters set new eyes on the science and the history of the science. Now that we know about jump-diffusion and stochastic volatility and discrete hedging and transactions costs and incomplete markets, and now that the actual history of the science has shown us the *necessity* to know about all this, how could a thin coincidence such as perfect replication under Brownian motion and the analytical tractability of the Black-Scholes model matter any longer? How could such a contingent fact even strike us as something worth mentioning in our rewriting process? History may originate from a degenerate case, but the history of a science, in the sense of the philosophical rewriting and grounding of the science, may not.

The trouble with Black-Scholes, however, is that history (real history, not the philosopher’s) could not have been written otherwise, and perhaps this singular fate is the most interesting part of the interpretive story. Indeed, how could Black and Scholes resist publishing their paper, and how could the public not welcome it instantly⁴, when it allowed the exact pricing (and hedging) of European options, and freed the valuation of contingent claims from the question of risk preferences? And how could option traders resist talking of implied volatility instead of option price, when Black-Scholes had shown them how to get rid of any other determinant of value through delta hedging, and left them with

volatility as the only measure of cheapness and dearness of options? Or rather, once delta hedging had eliminated first-order market risk, the option trader was left with a sense of option cheapness and dearness directly related to the risks he knew Black-Scholes *could not cover in reality*: gamma risk and vega risk. And here you can see the creation of Black-Scholes starting to act contrary to Black-Scholes. For what did the option traders do once they got hold of the Black-Scholes formula and measured the ease with which it allowed them to connect the value of an option and a volatility number? Create volatility smiles! So what Black-Scholes has done in the end is provide the option traders with the best way to talk and to act outside Black-Scholes!

The option language

And what would it matter anyhow if traders spoke an unruly and “un-regimented” language? Isn’t that always the case with natural languages? Accusing the traders of inconsistency on

Is it not precisely the lesson of the philosophy of language that logic shall not be the judge of language but the other way around ...

grounds of their multi-volatility talk is the same as arguing that every competent speaker, in every natural language, can sooner or later be forced into a contradiction, if the questioner pushes her strictly from antecedent to consequent and the black and white logic of truth tables is applied to her utterances. Is it not precisely the lesson of the philosophy of language (at least after Wittgenstein) that logic shall not be the judge of language but the other way around, and that both the notions of logic and “matter of fact,” so dear to the heart of the empiricist, shall themselves be relative to a language? Must there not be, as Richardson says, “a structure inherent in any language that provides the framework within which that language can *first* express any mat-

ters of fact”? Is the whole notion of ‘matter of fact’ not itself “internal to a logico-linguistic framework”? And “but for a prior specification of a logical structure,” wouldn’t the very notion of ‘fact’ be itself without sense? Language is robust precisely in the sense that one should not hold reality (or logic) fixed and try to vary the propositions of the language in order to come up with a falsity (or a contradiction) which would invalidate the language. On the contrary, one should hold language to be valid no matter what – for it is language that makes the world not the world that makes the language – and come to accept the fact that the contexts of utterance and their background logic may themselves be changing, in a word, that the world may itself be changing and that every speaker may be tacitly aware of it, every time some surface utterance strikes one as false or other-worldly. Language is not true or false, and it is not supposed to be a faithful picture of the facts. “Our words do not carve up nature at the joints” and nature does not care how many tenses we may conjugate our verbs.

Language is robust in the sense that it allowed us to travel safely through our thousands of years of evolution and to survive its many changing worlds. It is robust in the sense that we are able to have revolutions which overturn our most deeply entrenched conceptual schemes (such as Gödel’s theorem, Quantum Mechanics), yet we make sense of them *with language*. It is robust in the sense that we are able to do philosophy, to be reflective, etc.

Black-Scholes is valid and robust precisely in the sense that natural language is. Once we agree that what is meaningful and significant in Black-Scholes does not lie on the side of the lognormal assumption and the Black-Scholes formula – not on the side of complete markets and perfect

attainability of the contingent payoffs – but on the side of the dynamic relations that Black-Scholes has helped establish between the option, the hedge, the implied volatility representation and the movements of the underlying, we stop thinking of Black-Scholes as a theoretical model and start thinking about it as language. So long as the trader knows what he is doing, it doesn’t matter whether he changes the implied volatility parameter between two option trades, or between two delta readjustments. He is competent in that language. The option has first to exist, and second we have to start thinking of hedging it. It is the privilege of no option pricing model to bestow existence on options, even less so to rob them of their existence like the theoretical Black-Scholes does. No option pricing model⁵ is even entitled to establish the prices of the vanilla options in place of their own market. We’re not even sure that a smile model may be entitled to price the *exotic* options without somehow relying on their own market. All an option pricing model is welcome to do is provide the trader with a language, or in other words, a coherent way of travelling across the vagaries of the option world and of surviving its overturns. A language: that is, a conceptual scheme, a *Weltanschauung*.

And this general remark applies to the Black-Scholes model as well! Not the theoretical, vacuous Black-Scholes, but the meaningful, critical⁶ Black-Scholes. There is indeed a sense in which Black-Scholes is the first smile model! Don’t the option traders speak of Black-Scholes implied volatilities, and use Black-Scholes hedges, in real life option markets? And aren’t they confident of what they’re doing because they know everybody speaks the same language? The only practical use of Black-Scholes, after all, is to let you travel from point A to point B. And you are basically OK travelling with Black-Scholes so long as the delta (possibly adjusted to account for the change in implied volatility) takes care of first-order market risk⁷, and so long as you are confident that everybody will still be speaking the Black-Scholes language at point B (having made the same implied volatility correction that you did). Black-Scholes, and for that matter any option pricing model, are only here after all to ensure safe travel

Delta is the philosophically fertile notion and the entry point to all the different strands we've been exploring

through a price difference, not to quote an absolute price. Physics is essentially differential. And the key concept in every option model should be the option delta, not the option theoretical value.

The option delta

Delta is the *critical* concept here, in the two senses of the term. The trader's risk *critically* depends on the delta of his option position; in other words delta is the one important variable he will have to worry about after the inception of the trade. And delta is a critical concept in the sense that the entirety of our philosophical *critique* of option models has hinged on it so far, from our first contending that the smile problem really begins with the problem of the delta (or equivalently the problem of barrier option pricing), to our firm belief that the rule of delta hedging and rebalancing is the dispenser of scientific objectivity, to our conclusion that Black-Scholes is right and valid and meaningful to the extent that the Black-Scholes delta *should not* make the option redundant. Delta is the philosophically fertile notion and the entry point to all the different strands we've been exploring. First of all, it is the delta hedging idea which has made the language of implied volatility effective. Second, you can look at the delta from any side you wish, depending on your philosophical inclination. When it is part of the Black-Scholes derivation and formal theory is your sole concern, delta hedging leads to the strict option pricing formula that you know: it gives you the *law* that option prices obey.



When it is viewed against the neo-Kantian background of relational concepts and the priority of objectivity over truth, delta embodies the operative *rule* which *conceptually determines* the option. When it is reinserted in the pragmatic context of actual hedging, which necessitates a real time trader and his actual sense of opportunity, delta is your pathway to freedom: you can decide to over-hedge or under-hedge, optimally hedge, hedge discretely, not hedge at all, etc.

All of this hints at the idea that, once the options and their market are given and firmly given (contrary to their evaporation by Black-Scholes magic), we should first and foremost preoccupy ourselves with the hedge. Hedging is the key; option value is only a derivative notion. As for the option price, it is the purely opportunistic, almost political, variation of the option value. Hedging is the critical concept. For instance, we will show later that proposals to correlate default risk with the process of the underlying equity, which may sometimes go as far as invoking grandiloquent structural models of the firm, have as sole motivation the ability to produce higher equity deltas for the convertible bonds than in standard models, or indeed to generate such deltas for the straight debt, exactly like the trader would expect in real life. In this case as in many others, it is matching the delta that is the heart of the matter. Nobody really cares about the full underlying process, or the even less observable capital structure of the issuing firm.

REFERENCES

- 1 We are here reiterating the neo-Kantian view of concept formation. In Alan Richardson's (1998) words: "Perhaps the most important aspect of the neo-Kantian project [...] is the lesson it took from the development of pure mathematics and mathematical physics in the nineteenth century. For the neo-Kantians, this development exhibits a new type of concept formation that makes evident the functional nature of objective concepts and stands opposed to the traditional notion of concept formation via the process of abstraction."
- 2 Again, we are echoing the neo-Kantian view of scientific objects as individuated via their relations to one another. They neither are bundles of subjective impressions (following the philosophical doctrine of idealism) nor pieces of an absolute reality (following the philosophical doctrine of realism). "This view, writes Richardson, clearly contrasts with any naive realism that speaks of objective knowledge as objective not because of the systematic interrelations of the objects in the system but by relations to transcendent objects outside the system. Similarly, it is inconsistent with any idealism that founds objectivity in the subjective experience of any one individual, or that denies objectivity to knowledge in general."
- 3 Steven Galt Crowell, Husserl, Heidegger and the Space of Meaning, Northwestern University Press, 2001.
- 4 I am being guilty of history-rewriting, even here. For it appears that Black and Scholes had difficulty getting their 1973 paper accepted for publication. But this serves my interpretative point exactly. What I have called the "significance" and the "meaning" of Black-Scholes was not first apparent to the editor's eye. He could not have guessed the history that was to follow – the history of "volatility trading" – and the generations of volatility traders that were to come, from what looked, on the surface, like a simple analytical formula. In a word, he could not have guessed about the later philosophy of Black-Scholes, the part which came after Black-Scholes and that we have aptly identified with the smiles. Like I said, the "fact of science" in Black-Scholes does not belong to the 1973 Black-Scholes.
- 5 From now on, 'option pricing model' will mean 'smile model,' because we said Black-Scholes shouldn't really exist and smiles are the only thing there is.
- 6 Critical in the sense of the Kantian critique of metaphysics, and the subsequent construction of the objectivity of scientific theories.
- 7 Of course you will not be OK if jumps in the underlying occur between A and B. But we group jump risk under "gamma risk" and it is second-order in this sense, not in the sense of the magnitude of the loss.