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A Beginning, in the End

The concluding part of the discussion on a new view on quantitative finance

We don't think the money is what is so special about Black-Scholes and quantitative finance. Rather, the reason why we face this peculiar situation in quantitative finance, a situation where there is no choice but:

(a) To propose and to *have* a particular quantitative model, with its assumed dynamics and its embedded theoretical dynamic hedging (Black-Scholes-Merton in Taleb's and Haug's case)

(b) To go ahead and provide an exhaustive list of greeks and higher order greeks which are literally unthinkable within the confines of the theoretical model given in (a) (for instance, vega is unthinkable under the Black-Scholes assumption of constant volatility)

(c) To stop there and refrain from proposing a superior theoretical model which would systematize the risks hinted at in (b) and would make sense of the corresponding greeks (for instance, proposing a stochastic volatility model)

We think the reason why the "critical and all-encompassing model" will always have this hybrid nature and will always combine a closed and finished "piece of model" and an open platform, precisely exceeding the model, where elaborations like Taleb's and Haug's can find their place and proliferate, is the inherently *indexical* nature of option trading (or for that matter, of any trading). As we have said many times, Taleb and Haug are not so much describing a "definitive model" as they are describing a definitive skill, akin to language proficiency, that a competent trader ought to have with a given model.

Wanted – dead and alive

And the competence is kept open on purpose. The list of greeks and higher order greeks has to remain a list waiting for the competence of a trader to make sense of it as an operative whole, not for a systematic account to close it off and cast it into a higher order model. The reason why each particular greek is so valuable as to deserve a name of its own is not the value of the money involved or the rate at which it changes hands. It is because there corresponds to each greek a market dynamic precisely exceeding the prediction of the nominal Black-Scholes, hence a trading situation that the trader will have to face, and resolve, in actuality. The reason why Haug's written account stops at the list, and why he enjoins the trader to know the greeks and to take over where the theoretical model leaves off, is that the trader will have to make trading decisions in actuality, and actuality is not of such a nature as to be captured by a theoretical stochastic process or the option pricing model corresponding to it.

Actuality is not of such a nature as to be captured by anything written. We define actuality to be that solid block of reality precisely facing, and almost contradicting, any theoretical representation or attempt at a theoretical representation. (Here, you may start sensing an affinity between our "actuality" and Taleb's "central problem of risk management." The full correspondence will unfold later, however.) While your option-pricing model may predict a certain negative gamma somewhere down the road, it is a completely different story to actually go down that road and find yourself facing the negative gamma in actuality. It is so, despite the fact that the two gammas may be literally the same! "There is a difference between knowing the path and walking the path," says the character Morpheus in *The Matrix*. The reason why the greeks are so valuable is that the trader will have to spend actual breathing and living minutes inside the "gamma hole," minutes packed with the density, the singularity and the irreproducibility of a decision-making process not otherwise accountable or even representable. We insist on gamma because convexity is typical of option trading and of the risks attaching to it. Also, it is well known that time decay is the dual notion to convexity. So, parallel to the distinction between the gamma represented by the model (let us call it "theoretic-gamma") and the actually-experienced-not-otherwise-representable-flesh-and-blood

gamma (let us call it "gamma as of now"), there is a distinction between two aspects, indeed two philosophical notions, of the temporal. "This distinction was implicit in classical philosophy" but the Cambridge idealist philosopher John McTaggart Ellis "was the first to make it explicit and to give it name," according to Palle Yourgrau[1]. It is important that this distinction be made clear because it will shed light on what we have called "the indexical nature of option trading," so we will cite the passage from Yourgrau's book in full:

"McTaggart makes a considerable contribution to the analysis of classical accounts by introducing a fundamental distinction between two aspects of the temporal, which he calls the 'A-series' and the 'B-series.' [...] The B-series provides the fixed, immutable, indeed geometric relations between events in time in terms of *before* and *after*. The system of dates of events is B-theoretic, as are typical issues in the physics of time that are mathematically formulable [...]. The A-series, by contrast, characterizes events in terms of their shifting realizations as future, present, as past - that is, as *now*. Unlike the case with the B-series, an event's 'location' in the A-series is not fixed once and for all but is always in flux due to the *nunc fluens* – the 'flowing now.' Thus, whereas it is a B-theoretic fact that I am writing this in 1991, it is an A-theoretic truth that it is now 1991, so that I am writing it now. The former will still be true fifty years from now; the latter, however, will not[2]."

Terms like "now", "here", or indeed "this" or "I", are known by philosophers of language as "indexicals" and they are so distinguished because "they pick out different objects or places or times in different contexts of utterance. So your utterance of 'I'm hungry' picks out you, while my utterance of the same sentence picks out me" (*Oxford Companion to Philosophy*). It is really meaningless to ask who is "I", what time is "now" or what place is "here", independently of the particular context where such terms are used. The relation with option trading, as epitomized by the gamma's dilemma ("Shall I rebalance my delta now or let time fly?"), is that it is meaningless to try to model, theoretically represent, or even talk about, trading, independently of the actual context of trading. We cannot talk about trading in the third person mode. Trading, which is another word for "actualized decision-making," is always trading as of now, and admits "I" as only subject. We cannot talk about trading from someone else's point of view, let alone from nobody's point of view. (This agent-free, completely detached, point of view, also known in philosophical circles as *the view from nowhere*[3], is still perceived by many as the epitome of theoretic representation and as the ideal of objective science). Results of trading, money made or lost, closing prices of traded assets, charts and time series, etc., may be related and read in a newspaper (B-series), or indeed re-created in theoretical models and theoretical stochastic processes. Actual trading may not (A-series).

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The indispensable element of actuality

Accounts like Taleb's *Dynamic Hedging* and Haug's *Know Your Weapon* are complete as they are because they try to equip the trader with the knowledge and the tools he needs and they stop precisely at the doorstep of the *now* beyond which the trader alone can face his choices in actuality. They do not attempt to invade, that is, to remove, the preserve of the now by massaging its essential novelty into a new theory of risk and a new stochastic model. Paradoxically, it is such an over-theorizing temptation, for instance the temptation to complete and legitimize the decidedly "unfinished" and "unscientific" picture that Taleb and Haug have left us, by means of a stochastic volatility model re-embedding their vega partial derivatives, which would be in the last instance incomplete. For it would be ignoring the trader's place, and trying to bypass actuality (unless the new model is conceived of as the evolution of the weapon, and the trader is given new training to know this weapon again and master its ins and outs). What we are really saying is that, given the last component of what we have called the "critical and all-encompassing model," the item that we have identified as *the required presence of the actual living trader* and that Taleb echoes himself, in his complete account of dynamic hedging, under the heading of "the awareness that distributions are unstable and hard to model," it becomes a relative and almost minor affair how evolved and complex the given theoretical model is. Once the trader is equipped with the weapon and with the knowledge of the weapon, or in other words, once the theoretical representation is completed with the element of actuality as embodied by the trader's irreplaceable presence, all the models become equivalent. It may be a little harder to know the ins and outs of a stochastic volatility model than to know the ins and outs of the Black-Scholes-Merton formula. But this becomes an internal episode, and is the only reason why Haug claims superiority of the latter. There is no doubt, however, that a trader knowing the ins and outs of a stochastic volatility model will beat his Black-Scholes-Merton homologue.

It seems then that our disagreement with Taleb is over a subtle, though important, philosophical point. He essentially describes the trading situation as a case of decision-making *under uncertainty*. Not only are we not certain about the next market move on account of its random, or risky, nature, but we have no certainty or definitive knowledge about the random generator itself. Essentially uncertain situations, says Taleb, are situations where "no probability can be mean-

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ingfully assigned to possible future results." We, by contrast, would like to describe trading as decision-making *under actuality*. The problem with Taleb's philosophical position is that it still occurs on the same epistemological plane as the position of the risk managers he is criticizing. He may be opposing their views, he may be claiming that there might not even exist a generator of a certain general type in the first place, such that they could, in the second instance, try to guess its parameters, the fact remains that Taleb presupposes the category of existence (or non-existence) of the generator, and consequently deploys the corresponding epistemology and the corresponding skepticism. Even though he denies the existence of the generator, or at least epistemological access to it, Taleb is in point of fact committed to the whole ontological language.

Essential uncertainty and "hidden variables"

To be more specific, let us first review the move that a typically optimistic risk manager or risk modeler will make when she is faced with the bad situation Taleb is describing. If the random generator is indeed itself randomly changing, she will try to rise to a higher state space and assign probabilities to the different types of generators that may prevail. She will start talking about probability of probability. This amounts merely to generalizing the initial probability distribution and is analogous to the optimistic move from Black-Scholes to stochastic volatility, to stochastic volatility with asset jumps, and further to stochastic volatility with asset jumps and volatility jumps, as already witnessed in the several nesting cases that we have encountered. We understand from Taleb's radical pessimism, however, and from the meaning of essential uncertainty, that such an evasive action is not really available. Taleb rightly points to a fundamental gap between the notion of uncertainty and the notion of risk, and there is no filling this gap by "probabilizing" uncertainty, as it were, and by talking about the chances that the generator may not be what we expect (or in other words, about risk) when we should really be talking about the uncertainty of knowledge, period. The regress has to stop somewhere. We cannot just go on and assume the existence of a random generator hidden behind the random generator, so on and so forth. The distinction between risk and uncertainty cannot occur within the epistemological circle, and when it doesn't, it cannot be recovered by an appropriate expansion of the epistemological circle. It so happens that knowledge is concerned with risk and probability in our specific field of quantitative finance, but this is no reason why the picture should be allowed to spill over the frame, and uncertainty, which is constitutive of the very category of knowledge, become connected with its specific subject matter.

The optimistic move, trying to find a hidden random generator behind the random generator and trying to get away with uncertainty through the elevation of risk, really reiterates the same philosophical blunder as the so-called "hidden variables interpretations" of quantum mechanics. The reason why the epistemological circle cannot be closed in quantum mechanics is not just that it remains an open question where to establish the cut between what to count as object system and what to count as measuring apparatus. It is not just that the given wave function can always be generalized so to include both the object system and the measuring device, thus making it a requirement that a further apparatus be provided in order to "measure the measurement." For that would be equivalent to our infinite regress from the random generator to the generator behind the random generator, etc. Rather, the fundamental reason is that the measuring apparatus is constitutive of the meaning of the wave function, yet at the same time it qualifies as a physical object which may become an object *for* the wave function, like any other. Indeed, the wave function pertaining to a given object system is just an algorithm for computing the probability that a given apparatus may end up in a given definite value state after having interacted with the object system. As such (and following Peter Mittelstaedt), it provides "an interpretation that relates the theoretical terms of the theory to experimental data." The measuring process, says Mittelstaedt, is "part of a *metatheory* M(QM) that contains the semantics of the object theory in question, i.e., quantum mechanics," yet at the same time it is "a real physical process, and as a physical process it is subject to the laws of quantum mechanics." This results in a tension on the methodological level. Since they "serve as means for establishing a semantics and an interpretation, which provides a relation between object-theoretical terms and experimental results," the measuring apparatuses cannot belong to the domain of reality of the considered object theory. They belong to the metatheory. "On the other hand, Mittelstaedt goes on to argue, if quantum theory is assumed to be semantically complete, then the measuring apparatuses, considered as physical objects, belong to the domain of reality of the quantum object theory and are subject to the laws of this theory[4]."

The central problem of risk management and the problem of measurement in quantum mechanics

The same way that the quantum mechanical measuring apparatus falls on either side of the epistemological divide, depending on whether it is interpreted as part of the meaning of the theory or as part of its domain of reality, and that

there is no way of connecting the two sides, risk and uncertainty fall on different sides of the epistemological divide in quantitative finance and there is no way of reducing the one to the other. "Uncertainty of knowledge" cannot be rearranged into "knowledge of risk". The disconnection is stronger than physical separation or logical incompatibility. We are here dealing with two different philosophical categories. And the same way that the problem of measurement occurs in quantum mechanics because it is the same measuring apparatus that has this twofold role, so we are never sure, after the measurement process has taken place, whether the apparatus has fulfilled its semantic mission and definitely impressed empirical reality with the pointer reading of some eigenvalue of the theoretical wave function (the so-called "collapse of the wave packet"), or whether it has to be counted as a theoretical quantum object still enjoying the full superposition of eigenstates[5]. The central problem of risk management occurs in quantitative finance, because it is the same knowledge we are talking about, yet there are two different philosophical notions pertaining to it: its content (and this would be asking what it is the knowledge of, in this case, risk), and its form (and this would be enquiring about its conditions of possibility or impossibility, in this case, uncertainty and absence of definitive knowledge). And what is so peculiar about quantitative finance is that it is the same money that we are liable to lose, either as the natural consequence of (managed) risk or as the terrible consequence of a failure of our theory of risk. (It is because of this, because of the inescapable confusion of the result, that we see the door opening up to abyssal loss and abyssal epistemological difficulty in quantitative finance, not because of the unboundedness that Taleb is talking about. Compare the situation in the rest of the empirical sciences. Can you argue about "theory change" or 'paradigm shift' before the investors in your fund? And will they give you a break the same way that the history of the science usually gives the science a break at the time of scientific revolutions?)

True, you can always start the infinite regress from the measuring apparatus to the measuring apparatus measuring the measuring apparatus, and from the wave function of the object system to the wave function embedding both the wave functions of object system and apparatus, etc., or you can start the regress from the random generator to the random generator behind the random generator, etc., only this would be varying the internal episodes and would bring nothing philosophically new. On the contrary, the only interesting philosophical conclusion is to recognize, once and for all, the fact that the form / content duality is not eliminable when ultimate theories, such as quantum mechanics, get formulated at levels of generality so embracing that both the theory and the metatheory become part of the philosophical reflection, and to realize that the philosophical account of the given theory, or in other words our understanding of it, can no longer dispense with an element of actuality supplementing the theoretical, written account.

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Written theory vs. the actuality of context

If there is no telling, by means of pre-written theory alone, whether the apparatus is inside or outside the object domain, whether it is a theoretical object or a meta-theoretical ingredient, and if it takes the presence of an actual experimenter to establish that distinction and to interpret, in the actual context of experimentation, the right sides of the right epistemological divide, then the conclusion is that, whenever we are dealing with theories so ultimate and so refined as to call their own epistemological presuppositions into question, we cannot dispense with the specificity and actuality of our own situation as users and interpreters of the theory. A theory cannot become self-referential and self-critical without first including ... a "self." The main philosophical conclusion of quantum mechanics is the strong dependence of our way of understanding it and of making sense of it in the context of usage and experimentation. Rather than getting acquainted with the ultimate furniture of the world, the first teaching of quantum mechanics is getting acquainted with our own, un-exchangeable, situation as actors and interpreters of our world. A completely detached, agent-free, account of quantum mechanics will always be incomplete. "Just as it is meaningless to ask who is 'I', what time is 'now', what place is 'here', independently of the context of usage of those terms, it would be meaningless to try to specify an 'actualized experimental result' [in quantum mechanics] independently of the particular situation of he who takes part in the whole process known as 'measurement'," writes Michel Bitbol[6].

Likewise, we think that the only interesting conclusion in the methodology of quantitative finance is to recognize, then to fully endorse, the same kind of methodological tension as occurs in quantum mechanics. The reason why quantitative finance is so special is not the money involved but its peculiar epistemological set-up, once it is recognized that the methodology of risk cannot dispense with the "last component" that we have identified in any comprehensive model of risk, namely the actuality of the context of trading and decision-making. What we have called the "central problem of risk management," the fact that model risk is the biggest risk for a model of risk, is not really the coincidence that the play on words suggests it is. It is not a vengeance either. It is not as though risk had always had a way of punishing us and always used it after a while. When Taleb proclaims, in the announcement of the Wilmott–Taleb semi-

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nar, that "what can go wrong will go wrong," he is not just describing a special kind of malediction attaching to a special group of people known as risk managers. The fatal necessity he seems to be expressing is not the afterthought of some kind of demon, in charge of driving risk managers out of business. It is not as though you first had to come up with some risk management framework, where something, of course, could always go wrong, and then, second, this something went wrong. The necessity is all packed, from the beginning, in the very meaning of our science. As soon as you commit to the knowledge (or the science) of risk, you commit to the uncertainty of that knowledge. Not because of the pervasiveness of risk. For it is not the same risk that you attempt to study on the one hand, and that strikes back at you on the other. Remember the categorial distinction between risk and uncertainty. But it is because we positively wish to reach, with quantitative finance, an extreme and limiting case of an epistemology, the same as with quantum mechanics, where knowledge has both to admit of an object, in this case risk and the given theory of risk, and to be subject itself to a metatheory, or at least a meta-critique, the one that deems it uncertain.

If quantitative finance didn't exist, we would have to invent it in order to create an opportunity for such an epistemological limit and an opening for the corresponding methodological tension. And what imposes on us such a conflation is the peculiar position of quantitative finance among the other sciences[7]. Quantitative finance is no ordinary science because it is not just concerned with knowledge and the passive accumulation, or structuring, of knowledge. Nor is it just concerned with experimentation and the subsequent verdict of empirical reality. Remember the entanglement we already pointed out. Quantitative finance is the enlarged science in charge of explaining why traders keep using and, as a matter of fact, should actively be using, quantitative models which are plainly false by the lights of the lesser form of science. This necessary excess and constant withdrawal (to use a term from Heidegger) cannot be covered by an appropriate expansion of the representational framework. This is the reason why actuality is imperative and pre-written theory can only fail. This is why Taleb's *Dynamic Hedging* and Haug's *Know Your Weapon* are finished written accounts, yet at the same time are un-finite, that is, completely open, writing pieces.

Generalizing probability theory

Taleb's choice of the word "uncertainty" (which smacks of the representational schema) would apply if the game had to stop at passive knowledge and quantitative finance, like the econometric examples he cites in his paper, was only concerned with statistics, the identification of random generators, and the estimation of their parameters. Taleb's critique and subsequent skepticism would apply if the only point was an epistemological point and if traders were searchers whose action – or wisdom, or instinct, or science, or philosophy – did not precisely exceed their knowledge. And yes, Taleb is right in pointing to the situation of essential uncertainty in quantitative finance, and in blocking the optimism of the traditional risk manager. But is he thereby recognizing anything specific to quantitative finance, anything not readily covered by the Knightian definition of essential uncertainty? Taleb always sounds as if he is breaking some philosophical news to the philosophically uneducated risk manager: problem of induction, essential uncertainty, etc. But has he ever thought about the possibility of breaking philosophical news, one day, to the financially uneducated *philosopher*? What if the critique had to go the other way and the situation in quantitative finance had to provide the lead for a new kind of philosophical and epistemological thinking? We hear that Taleb is preparing a "treatise on probability": Keynes revisited at the age of derivative instruments and derivatives blow-ups. Shall we expect the author to cover new and original topics, or will he be reiterating the classical story, from subjective probability to objective probability, and from risk to uncertainty, only modernized and revamped?

Speaking of probability, the received view about quantum mechanics is that it is an ultimate physical theory making essential use of probability. The most challenging interpretation of quantum mechanics, however, goes exactly the opposite way. It argues that quantum mechanics, quite independently of its celebrated application to elementary particles, is itself the generalization of probability theory to cases where we cannot abstract away from the context of experiment and where the range of possible outcomes has yet itself to be determined, before we start asking what particular outcome will obtain with what probability. Quantum mechanics is not just a physical theory. It is at the same time a metatheory of knowledge and of probability. According to that interpretation, the wave function and its unusual "interference of probability term" in fact just reproduce the algebra of a general "theory of predictions" which becomes our last recourse in those general cases where contexts do not necessarily commute and the particular order of the experimental protocol may matter[8]. It so happens that experiments at the macroscopic level occur far enough from the epistemological limit[9] for us to even notice the

context in the usual cases, and to start suspecting the fact that properties may not be borne by objects independently of the context of observation after all. And it so happens that elementary particles are tiny enough and "fine" enough for that epistemological limit to be finally solicited. So in a sense, the elementary particles are the ultimate constituents of matter, not just because the physical process of dividing up matter in ever tinier pieces has to stop somewhere, but because the epistemological process of bracketing the context and focusing only on properties supposedly inherent in the "object," a process that occurs, formally unchanged, every time an epistemological invariant is about to crystallize into an object (of knowledge), has itself to stop at some point.

A philosophical divide

The fact, brought up by quantum mechanics, that the presuppositions of our system of knowledge should come to the surface at some point, and that questions like "What do we exactly understand by the word 'observation,' and the word 'fact,' and the word 'property,' even the word 'object'?" should themselves come to light at the same time as we conduct an experiment presupposing their answers, is indeed a very welcome fact. And the fact that our philosophical thinking should proceed, at its finest level, not only as a metatheory of knowledge and epistemological critique, but also as a critique of language proper, is no coincidence. Like Bohr has pointed out, quantum mechanics is no remote or esoteric science, and it is an a *priori* requirement that we should be able to talk about it, and give accounts of its phenomena, with our ordinary language. Quantum mechanics is closest to our language than we think, because it is close to a definition of our scientific language and of our expectations about it. Is it surprising, really, that the theory of elementary particles should at the same time command an elementary theory of language? Quantum mechanics is ultimate in many ways and initial in many ways. When it appears to have definitely broken the chain of "theory embedded in written theory embedded in written theory," and when it demands, for the first time, that the context of writing be brought to the fore and itself made thematic – thus bringing in a fundamentally new element of actuality – we understand that quantum mechanics can be seen by some people as a hopelessly "incomplete theory" (Einstein) and by others (Bitbol, etc.) as a fundamentally complete *theory cum metatheory cum philosophy of science*.

Here lies precisely the philosophical divide between the second Taleb and us. While *Dynamic Hedging* had offered him every chance of "completing" the science and carving the possibility of the science out of the necessity of philosophical thinking about the situation of the science, Taleb has preferred, in his second book, to lapse into denial and skepticism. According to our interpretation, *Dynamic Hedging* was typically the sort of book to have recognized and endorsed the "A-character" of option trading ("A" for A-series and for Actuality). By stopping short of re-embedding the ins and outs, or the rights and wrongs, of the Black-Scholes model into a higher-level theoretical model and by solemnly declaring that no experienced trader would in effect and in actuality "trade Black-Scholes-Merton for another pricing tool," by thus putting an end to the temptation of theoretical rewriting and by fully acknowledging the irreducibility of actual experience and actual decision-making, the first Taleb makes a perfect case indeed for the distinction between what we shall call: *stochastic process in representation* and *trading progress in actuality*.

FOOTNOTES

1 Palle Yourgrau, *The Disappearance of Time*, Cambridge University Press 1991.
2 Palle Yourgrau, op. cit. p. 27.
3 Thomas Nagel, *The View From Nowhere*, Oxford University Press 1986.
4 Peter Mittelstaedt, *The Interpretation of Quantum Mechanics and the Measurement Process*, Cambridge University Press 1998, pp. 4-5.
5 Erwin Schrödinger has given a lively illustration of the problem of measurement in quantum mechanics with his famous dead or alive "cat paradox."
6 Michel Bitbol, *L'aveuglante proximité du réel*, Champs Flammarion 1998, p. 294.
7 Here you might object that skeptics like Taleb are not even granting quantitative finance the label of science, so how can we talk about its position among the sciences? Only you have to remember our positive philosophical drive and our special brand of philosophy of science, especially when it is "applied" to a science, like quantitative finance, which is still in the making. When there is no science and there is only skepticism about the possibility of the science, we wish to take the transcendental argument to an even higher level than the "necessity of the possibility." We shall create the science. We shall create a new brand of science for the sake of arguing philosophically, then scientifically, about that science. The opportunity is offered us, in quantitative finance, to proceed with the philosophy and with the critique before the identification of the actual science, then to bootstrap the science out of the philosophy of the science. That a philosophical critique like ours is possible is the best proof of the existence, therefore the possibility, of the underlying science. This is how far we wish to go with the transcendental turn. This is the unique opportunity that, we think, quantitative finance offers philosophy, and philosophy, in turn, will offer the science, once the science is identified.
8 I am here drawing from Michel Bitbol, *La mécanique quantique comme théorie des probabilités généralisées* (in: E. Klein & Y. Sacquin (eds.), *Prévision et probabilités dans les sciences*, Editions Frontières, 1998). Bitbol writes: In all the sciences, as well as in many ordinary situations, there usually corresponds to each perceptual or experimental context a given range of phenomena or possible determinate outcomes. For instance, to the context of the retina there corresponds the range of colors, to the context of the ruler there corresponds the range of lengths, to the context of the thermometer there corresponds the range of temperatures, etc. As long as the contexts can be conjoined and the outcomes remain insensitive to the order in which the contexts are brought into play, one is justified in merging the different ranges of possibilities (relative to different contexts) into a single global range relative to a single all-encompassing context, then in forgetting about this context altogether and in dealing with the elements of the global range of possibilities as if they reflected determinations inherent in the object under study.
9 This is the limit, made evident for the first time by quantum mechanics, where the measuring apparatus starts playing a twofold role, both inside and outside the object domain, in other words, the limit where the object of knowledge can no longer be independent from the condition of possibility of knowledge. (If you are starting to sense a transcendental flavor in our turn of phrase, see Michel Bitbol for an essentially transcendental interpretation of quantum mechanics).