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Probability as the Other Face of Knowledge

Uncovering a broader view of probability by taking a lead from Quantitative Finance

In the previous issues of *Wilmott* magazine, we established that Probability can be generalized to the meta-level where the whole range of possible outcomes – and not just a particular outcome – is not yet determined, provided the Boolean algebra of the classical Kolmogorov probability is generalized into an ortho-algebra and the rules of classical probability calculus are generalized into the formalism of the "wave function" and the underlying Hilbert space (as in Quantum Mechanics). Knowledge would then have to consist of two components: a classical *representational* component where the world is split in perfectly stable and identifiable "states of the world" and a *performative* component which is the knowledge, or rather the "enactment," of the actual context of experiment. As such, the latter is inseparable from the performative choice of the experimenter.

Applying this insight to quantitative finance, the claim would be that the given option pricing model (the particular "weapon," as Haug would say) and its particular representation of the states of the world belong to the representational side, whereas the "Know your weapon!" and the prescription how to handle its "ins and outs" belong to the performative side. This performative knowledge is inseparable from the trader's situation and actual role.

Only the pair composed of the weapon *and* the knowledge of the weapon, or in other words, the pair composed of the option pricing model

and the trader, can "know" the market. This knowledge, we have called "new knowledge." *Essential uncertainty* attaches to *representational* knowledge of course, and is dismissed as soon as representational knowledge is deemed insufficient. It should be replaced by *essential actuality* which attaches to new knowledge.

Still the question could be asked: How can we provide a *theory* of that new knowledge? What kind of epistemology is this epistemology where actuality is the only rule? Are we saying anything more in effect than that the trader must be ready for any contingency, and will know what to do in any circumstance, just because he is a good trader and because *he knows*? Is not our epistemology collapsing into flat pragmatism

and pure opportunism? What science can we possibly bootstrap from our blind trust in the trader's capacities? Again the answer will be that the ambition of our epistemology does not lie in the ordinary direction in which new theory can come and supersede old theory and new structure come and replace old structure. From the simplicity of our epistemological conclusion (that 'essential actuality' should replace 'essential uncertainty') we wish to draw a consequence, and hopefully a philosophical result, for large philosophical categories such as probability, and truth, perhaps even theory itself. We agree with Taleb that we may be facing, in quantitative finance, the limit of the traditional quantitative apparatus; only we wish to use the

limit in order to shed new light on probability, perhaps even establish its real meaning. Earlier we mentioned the "reinterpretation and the rethinking of probability" as one of our favourite consequences of quantitative finance. We even suggested that quantitative finance may be redefined as this revision of the notion of probability. Indeed we expect no less from our new epistemology than what we have gained from our quantum mechanical lesson. As with quantum mechanics, we believe that the first wrong statement is the one according to which quantitative finance uses probability theory. Looking for the random generator, or blocking epistemological access to the random generator, both attitudes presuppose that probability theory, in the traditional sense, is a proper (or improper) part of quantitative finance, precisely the part that lends it the "quantitative" adjective. Both attitudes in fact take a narrow view on probability, the one pursuing no greater an ambition than just finding a probabilistic *model*, and the other no greater a conclusion than the limitation of all such models. By contrast, we want to take the lead from quantitative finance and uncover a much broader view of probability than our previous epistemologies had thought. Probability is not just about stochastic processes and stochastic calculus. It is not just about transitions to one from numbers *strictly less than one*. It is not about possibility and actuality in the traditional sense. It may not even be about the whole set up involving risk, uncertainty, decision making, and the special brands of knowledge corresponding to each.

Rethinking probability

In a recent addition to *Fooled by Randomness*, Taleb rightly distinguishes between what he calls "probability thinkers" and "probability calculators." "The problem of calculus of probability," he says, as it has occupied most of the conventional discussions on probabilistic thought, is "hair-splitting for me." To his mind, the real problem of probability is a matter of knowledge, not one of computation. And the real question is: "Where do we get the probability from? How do we get the right assumptions? How do we make sure we are not gambling with the wrong dice?" *Thinking*

about probability is not so much a matter of developing a sophisticated probability model as it is investigating the role of probability in our overall theory of knowledge. Probability is the science of uncertain outcomes anyway, so you can go ahead and specialize *yourself* in the scientific methods of probability, in the ways of combining, compounding, or differently weighting, the different outcomes. You can become a probability technician, a "thoughtless probability calculator" as Taleb would say. However, when you start thinking about probability and its epistemological role, about what probability is and about its meaning, you no longer can draw a separating line between the subject matter of probability, uncertain knowledge, and the condition of knowledge of the probabilities themselves. The penetrating philosophical point that Taleb is making is that, once you start becoming reflective about probability, you can no longer ignore that "it is the same knowledge we are talking about," the knowledge whose uncertainty we are trying to quantify, on the one hand, and the knowledge whose *general theory* embraces probability as a whole, on the other. Philosophical reflection has to deal simultaneously with the object level and the meta-level. Probability, we may say, is a kind of theory of (uncertain) knowledge. And epistemology is also known as the "theory of knowledge." Does this put the two on the same footing? Think of the different meaning of the expression 'theory of knowledge' in the two instances. Taleb's point is that, when knowledge rises from the status of being an object for probability theory to the sta-

tus of being the knowledge of the probability itself – as should be the case in philosophical reflection on probability – the most important question becomes: "How certain is the latter form of knowledge? How certain is the knowledge of the probability?" And this, of course, opens the door to *essential uncertainty*. Essentially, the point Taleb is making is that, in order to really think about probability, one should skip all the computational details, and concentrate on the one big problem of *essential uncertainty*.

We for our part are pursuing a rethinking of probability, not just a thinking of it, and we wish our rethinking to take us precisely beyond essential uncertainty. However, we retain from Taleb's addition to his first edition the step from computational rationality (or Machenschaft, as Heidegger would say) to thinking. Taleb's addition is useful to our overstepping purpose in that it asks for the first time the momentous question: "What is thinking?", and shows the essential ingredients of the first step: the reflective attitude, the neglect of the object of probability – that which it is the probability of – and the turn instead towards the subject of probabilistic thinking, the cognitive agent whose general theory of knowledge and its conditions of possibility we wish to consider now.

If the first degree in the hierarchy of probabilistic thinking is to start wondering where we get the probabilities from (in other words, to start questioning the assumptions of the probability model and stop indulging in its computational delights), then the second degree on this reflective

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scale will be to wonder what the origin of probability itself may be. Taleb's *thinking* about probability has not yet gone all the way. True, thinking has turned critical for the first time at his hands and Taleb raises, for the first time, the issue of the *conditions of possibility* of knowledge over and above the technical issue of the partitioning of knowledge over the probable and the less probable. True, uncertainty (of the knowledge of probability) has for the first time taken precedence over probability (of certain knowledge). Still, Taleb is being only half reflective, for his critique and reflection are still playing *one* probability model against the other ("Before throwing the dice, we'd better check whether it is the *right* dice.") Or perhaps he is playing all probability models against the impossibility of finding the right one ("Perhaps the right dice does not exist.") Severe as the uncertainty may be, the conditions of knowledge that Taleb is considering still appear to be the conditions of knowledge *of* probability. Probability still holds the grammatical place of the object of knowledge. It has not yet itself interpenetrated the conditions of knowledge. It has not risen to the status of an indispensable precondition of knowledge.

Instead of wondering how certain the knowledge may be of a probability model which is meant in the first place to quantify the uncertainty of knowledge, and instead of wondering where we get the probabilities from in the particular instances, perhaps we should start looking, once and for all, for stronger *a priori* links between knowledge and probability. Imagine the last degree of epistemological and probabilistic thinking as the degree where the probability is no longer the probability of *something or other*, and knowledge is no longer the knowledge *of* probability, but where probability just becomes the other face of knowledge. The lesson from probability should be plain epistemological (no ontology involved: neither the ontology of events whose probabilities we are measuring, nor the ontology of the probability distributions themselves, which people believe exist in the world and believe we may get to know or never get to know). And the lesson from epistemology should be probabilistic (once the conditions of knowledge are really examined – and by 'really' we

mean that we should forever despair, as Pascal says, of "knowing the end of things and their principle" and should understand that "knowledge can never detach itself from the nexus of relations between things, but will always find itself caught in the *middle of things* and will always depend on the particular perspective" [1] – once the notion of secondary qualities thus takes over completely the notion of primary qualities, probability emerges as the true face of knowledge).

Probability as the precondition of intransitive knowledge

Indeed the major philosophical lesson from Quantum Mechanics is the epistemological (as opposed to ontological) reinterpretation of indeterminism and probability. (And we do not mean by 'epistemological interpretation of probability' the traditional view which imputes probability to ignorance and to the lack of *knowledge of something*; we mean something much stronger, something intransitive which has to do with the condition of knowledge itself). Indeterminism in Quantum Mechanics, as epitomized by Heisenberg's uncertainty principle and by the image of "perturbation of the object by the measuring device," is in fact due to the inherently contextual character of the quantum phenomenon, not to some underlying, subsistent, process that we can supposedly only investigate by perturbing it, therefore can only observe indeterministically.

In other words, it is the very condition of possibility of our *knowledge* and its manner of being which produce the indeterminism in those "fine cases" where knowledge depends on the context, not the lack of complete or determinate knowledge about some underlying ontology. As Bitbol writes:

"The image [of the perturbation] consists in first staging a universe of objects endowed with spatial and kinematical primary qualities, then in invoking their mutual alterations, in order to justify in retrospect the abandonment of the concept of primary qualities and its takeover by the concept of secondary qualities. The only reason one gives rise to the representation of a universe made of shapes and of motions through this image, is to show its uselessness. And this is equivalent, in a verificationist epistemology, to claim-

ing its inaccessibility in principle."

As a matter of fact, a formal link can be established directly between indeterminism and contextuality, without the ontological intermediary of the perturbation. Bitbol cites the work of Paulette Destouches-Février in this regard. Indeed, Destouches-Février [2] has shown a theorem, in the early fifties, to the effect that any predictive theory.

This is what provides, to our mind, the answer to the question: "Where do we get the (whole category of) probability from?" which was the last question on the reflective scale of probabilistic thought. Essential indeterminism, and the subsequent appeal to probability, *is the direct consequence of the dependence of the observable phenomena on contexts of experimentation which may be incompatible with each other and may not commute*. Probability is due to the formal condition of our epistemological apparatus. It is the consequence of the way our knowledge is structured and to the structural ingredients of our process of knowledge (experimentation, the context of experimentation, the general fact that the range of possible outcomes may depend on the particular context and may itself be indeterminate at the level above, the level of a general "theory of predictions," not mentioning that the causes of the phenomenon may themselves depend on the context of its production, therefore may not be defined before the phenomenon actually takes place![3]

Probability, we may say, is the direct consequence of the *form* of our knowledge, not of its *content*. What earns us the epithet 'essential' in 'essential indeterminism' (or 'essential uncertainty') is this a *priori* link between indeterminism and the *form* of our knowledge, not the fact that we shall find probability to be uneliminable from Quantum Mechanics or from quantitative finance for reasons inherent in the entities under study. Essence and necessity are here of formal-semantic, not of material-ontological, nature, and the link between indeterminism and the *form* of our knowledge is here the strongest form of link. It is not the case that indeterminism is inherent in the elementary particles or in the market, and that we cannot track the particle without perturbing it or frame the market without putting the frame at risk. Both Heisenberg's

and Taleb's "uncertainty principles" are figurative captions and the heritage of the classical representational schema which stages a world *facing* us, either transparently or opaquely, but at any rate epistemologically separate from us. They lure us into investigating the peculiarly "jumpy" nature of the particle (to try to explain why its spatial position and its momentum cannot be measured jointly), or investigating the "wholly complex" nature of the market (to try to explain why risk will always run one step ahead of the model supposed to run risk, and why the risk manager will always be running the risk of being run over by risk), when we should on the contrary become reflective and turn towards the general conditions of knowledge. No matter the content of our knowledge and whether it particularly attaches to elementary particles or to global markets, probability is among its prior conditions and relates directly to that, in knowledge, which is independent of the particular content, in other words, its structure.

Leaving the representational plane

In the end, we are asking Taleb to himself leave the classical representational schema and to embrace this higher view of probability. The traditional risk managers falling under Taleb's criticism may be happy with their frameworks and with their models. They may even be less naïve and more reflective than we think, and they may claim that their belief in their probabilistic models is only methodological and not realistic. "It is only by taking our theoretical entities seriously," they may argue, "that we can feel motivated to carry on our research program." However, we do not see any positive reason why Taleb himself should continue to worry about the classical view of probability and write about (the lack of) old knowledge. Is he not, in the end, himself bringing into life this "universe of shapes and motions," and himself speaking of random generators, for the sole purpose of showing the uselessness of such representations and the pointlessness of the corresponding quantitative research program? And does not the rest of Bitbol's assessment then equally apply to him, namely, that the image of the "perturbation" and of the random generator

kicking back revengefully at the quantitative analyst, is equivalent, *in Taleb's verificationist epistemology*, to the inaccessibility of certain knowledge *in principle*, therefore to essential uncertainty? What we are saying in effect is that Taleb's radical skepticism ("We don't know that the random generator is of a certain general type. We may not even know that the generator exists, that it is.") is the fruit of two mutually annihilating movements, one that first establishes the image of the random generator and one that demolishes it almost as quickly under the rule of verificationism.

We propose an even more radical step. Don't even start talking or thinking about the random generator! Semantic denial is stronger than existential denial. The slogan "There is no random generator" should in effect read: "There is no 'random generator' – there is no *talking* about a random generator!" Once the representation of the random generator is retired from the epistemological equation, the probability of outcomes supposed to be given by the random generator and the probability that the random generator may not be the right one fuse into the one big probability that is left, the essential probability that we owe to the (intransitive) structure of our knowledge and to its contextuality.

It remains to be shown, of course, that the markets do present us with such a case of contextuality, where a meta-contextual probabilistic tool is required and the contexts are mutually incompatible. Enough to remark, for the moment, that Taleb has no choice, after denying us any certainty we may have about the probability of possible future results, but to believe that

the ultimate random generator exists, only we will never get acquainted with it, or that nothing exists and even the ontological ground should give way under the theoretical edifice. In either case, the only reason why Taleb would want to remain in the representational plane will be a negative reason. His only business will be to counter every epistemological hope with the same radical skepticism. His invariable philosophical position will be "to sell epistemology short." Our radical step outside representation, by contrast, offers substantial epistemological upside. Recall that we are being even more radical than Taleb in that we don't even want to talk about the random generator and that we wish to relieve Taleb from the whole duty of first representing the generator and second declaring it inaccessible. We wish to remove from Taleb's philosophical picture the whole panel on which the image of the market was getting projected as an entity facing us (with our without accessibility to its random generator), and which was serving no other purpose, at the end of the day, than to *articulate* Taleb's skepticism. We wish to undertake an act of removal and amputation, so how could there be a gain in such a subtraction?

What we are losing quantitatively, we may be gaining philosophically. We may be losing theory and we may be losing the criticism of theory, we may be losing the opportunity to write second thoughts in second books like *Fooled by Randomness*, we may be losing the whole image of the market as an entity, open or hermetic, dwelling in front of us, we may be losing the science we were seeking before we have even grabbed it, yet in the end we gain, we gain in the *reflective* reorientation of our epistemology. As Bitbol writes:

"... The philosopher has really a lot to lose if he lets himself be absorbed only by the relation between theory and world. Indeed such an attitude does not encourage him at all to reflect upon what theory owes to the *situation of man himself in the world*, and in particular, what it owes to the very practice of experimental investigation. By contrast to the daily working scientist, the philosopher cannot satisfy himself with the Pascalian situation of man as perfectly integrated in the environment he explores; he has to

think this situation through and try to spell out its consequences. As a matter of fact, the scientific researcher himself may benefit from adopting the reflective stance from time to time, typically when the time comes for the reorientation of his work. And everyone knows that he almost inevitably has to adopt such an attitude during scientific revolutionary periods.

The unrepresentable process of "strong actualities"

Still, it may sound implausible that we should draw an epistemological analogy between Quantum Mechanics – which occupies Bitbol in the present instance – and quantitative finance. There can be no purer, no finer and no clearer-cut epistemological case than Quantum Mechanics, which deals with the basic constituents of the universe, and there can be no smokier and no "dirtier" case than quantitative finance, which deals with the multiform, most liberal and most complex dealings of the most evolved species of the universe. We have already warned the reader that he should not expect from this parallel that we may come up with the equivalent of the wave function in quantitative finance. When we say we wish to redefine quantitative finance as a generalization of probability theory similar to Quantum Mechanics, we mean it only metaphorically and analogically. While the generalized probability formalism is available in Quantum Mechanics, with the ortho-algebras replacing the Boolean algebras and the quantum probability functions replacing the classical Kolmogorov functions, no such formalizing is imaginable in quantitative finance. Yet we wish to retain from the quantum mechanical analogy the generalizing move rather than its formal product, and we want this move to carry over to quantitative finance. The same way that Quantum Mechanics has taught us to look differently at probability – if only because it opened up a higher level of generality where the contexts of the predictable phenomenon could not be mutually compatible and a generalized probability tool was required – we wish to look differently at probability *after* quantitative finance. The generality we are after lies in our epistemological relation to probability, not in the probabilistic formalism itself. Just as a whole new conception of probability was called for in Quantum Mechanics because the elementary particles put us in the extreme epistemological position where the contexts mattered at last, we pursue in quantitative finance a similar epistemological limit, that we have called "epistemological occasion."

Something constitutive of the category of probability may very well have to change after quantitative finance. Maybe the relation between possibility and actuality will no longer be the same. Maybe the novelty will be this, that the internal possibilities assumed by any of the stochastic processes which come to be given in quantitative finance will never come to pass as normal probability theory supposes they should, and the only actualities that will get actualized as a matter of fact will be the successive actualities of the *different, mutually incompatible contexts*, each one of which implies, of course, a whole change of the range of possible outcomes and of the corresponding stochastic process. If so, what overall stochastic process will possibly describe this "process of actualities"? Since the actualities we are talking about are the actualities of the contexts which get picked up, by definition, in a completely extra-theoretical way and completely outside representation, how could representation recuperate their process, and an overarching stochastic process be *written* for them?

Perhaps a stochastic process shouldn't be written. Perhaps this process of "strong actualities" should remain forever free from the recuperative attempts of writing. (This will pose the problem of writing at all the levels at which writing can take place, as we shall see later: the writing of the process, the writing of critical papers such as Bitbol's or Taleb, even the writing of our own present text.) In the last instance, probability was found to be the other face of intransitive knowledge. When transitive knowledge was found to depend on the particular context, and when it turned out, in the last instance, that the contexts could be mutually and irreducibly incompatible, *a general theory of predictions* had to come and sweep away our last hope in the possibility of an independent description. (When the contexts are mutually incompatible, the only available predictive theory is an essentially inde-terministic theory. You can no longer attempt to *describe* an object, or even represent it as a bearer of properties and a partner of epistemological intercourse; you can only try to predict the outcomes of a whole phenomenon where the domain of the object and the domain of the measuring agent are no longer separable). Essential indeterminism and essential probability, in the sense of the complete takeover of description by prediction, thus became the limit of knowledge. And now we stand, in quantitative finance, at the end of knowledge and probability. When we contemplate the "process" of strong actualities probability cannot enter the picture all over again.

Probability took us as far as the recognition of the fact that knowledge, in the last instance of its entanglement in mutually incompatible contexts, was only probable. Probability, we may say, is knowledge as *so entangled and so generalized*. This is the end of the meaning of probability. Its end: its finish and its purpose. So how could probability now overstep its own self, and get written for the one process precisely occurring outside knowledge, the process that we cannot know: this process of strong actualities *which change the contexts which make the ends of probability and of knowledge meet?* Are we not finally reaching the "epistemology" we announced earlier: "the epistemology where probability is eliminated, and where actuality is the only thing that counts"?

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FOOTNOTES

1 See Michel Bitbol, "La Mécanique Quantique comme théorie des probabilités généralisées", in E. Klein & Y. Sacquin (eds.), *Prévision et Probabilités dans les Sciences*, Editions Frontières, 1998. Also http://perso.wanadoo.fr/michel.bitbol/genproba.html 4
2 Dealing with phenomena whose experimental contexts may be mutually incompatible, is "essentially indeterministic."
3 Destouches-Février, Paulette, 1951. La structure des théories physiques, Paris: P.U.F. 3 See the discussion of causality in *Quantum Mechanics* by Grete Hermann (Grete Hermann, *Les Fondements philosophiques de la mécanique quantique,* Paris, Vrin, 1996).