What is Implied by Implied Volatility?

Elie Ayache begins a new series taking his hammer to some cherished concepts

Bruce Banner joins the financial world

Word and concept

Implied volatility is not just a word or a concept.

As a word, what is implied by implied volatility – what "implied volatility" means – is the value of the Brownian diffusion coefficient, inherent in the Black-Scholes theory, that you get by inverting the Black-Scholes formula for the theoretical value of a European option against the empirical market price of that option. From which you immediately see that the meaning of the word "implied volatility," as is always the case with meaning, will depend on the meaning, I dare even say the meaningfulness, the validity, of other group of words, perhaps theories or even whole conceptual schemes, in this case "Black-Scholes." We shall spend time later exploring the exact meaning of *this* – what this means exactly that, in order to fulfill the meaning of "implied volatility," one should make sure that the Black-Scholes formula *validly* implies it. It will all hinge, as you will see, on the notion of hedge. The Black-Scholes formula reposes on the continuous hedging argument. Therefore you have to first make sure that the hedging is validly conceived in order to guarantee that the Black-Scholes formula may imply something like "implied volatility." Continuous hedging is thus part of the meaning of implied volatility, not to say its presupposition. Consequently your next concern, after implying implied volatility, ought to be: "How was the hedging achieved already?" Or more specifically: "Under what volatility was the hedging achieved?" This question is all the more pressing that, by the very meaning of "implied volatility," there is a question mark hanging over the value of volatility.

At this point, all this may sound trivial and tautological to you, for isn't the meaning of "volatility" in Black-Scholes precisely the number that you should use in the formula, both for the pricing *and* the hedging? And could therefore be any question that this number should be used in the algorithm to compute the hedge and no other? Actually, the answer is far from clear, for you might legitimately wonder, given that hedging is not just the formal argument used in the formal derivation of Black-Scholes but has, among other material consequences, well, *hedging*, and the corresponding course of action that will unfold in time, whether real volatility will *actually* turn out to be the same as implied. For surely we don't live in the world described by Black-Scholes; and this is no accident, for isn't the true purpose of option trading, consequently of "implied volatility," to detect discrepancies in the market prices of options and try to make some money by buying or selling these options; that is to say, by arbitraging their prices against what will turn out to be their "true value"? And isn't this true value the reflection of the cost of the actual dynamic hedging that will ensue?

What we are witnessing here are the implications of "implied volatility" as *word* quickly developing into the implications of "implied volatility" as *concept*. While "implied volatility" as a word implied hedging under implied volatility – and no other number – by the simple logic, or meaning, of Black-Scholes (it implied it *analytically*, in philosophical parlance), "implied volatility" as a concept implies a lot more. With the concept of implied volatility, we step beyond the analysis of the Black-Scholes formula and the success or failure of inverting it. We question the validity, the significance, and the implications of such an act. We wonder what it may mean for the whole intellectual landscape lying behind the formula, and the whole scientific practice that is conditioned by it, that the number called "volatility" should be retrieved from the prices of options traded in something called the "market."

What I am suggesting here is that "implied volatility" is mainly a trading concept. For instance it can be taught in an option trading school and we can try and picture the consequences of such a teaching and such a program, i.e. the subsequent rule of action, the subsequent disciplined behavior. As a trading concept, implied volatility says: "From now on, you shall go and find in the option market itself, the answer to the question about that unobservable parameter that goes in the Black-Scholes equation." A lot is involved in that single step, not least among which the admission that the market may indeed act as an independent physical reality that we may confront with our theory and calibrate our theoretical model against.

Compare the other inverse problems that arise in the physical sciences. For instance, Young's modulus in solid mechanics is used to predict the elongation or compression of an object under stress. It is crucial in bridge design and the selection of the appropriate material. However, it is not an observable property of the material as it measures only a *disposition*. One way of determining it is precisely by re-calling its meaning, that is, by submitting the material to a mechanical stress of known intensity, by measuring the corresponding deformation, and finally by inverting the formula provided by elasticity theory.

There is an ongoing philosophical debate on whether dispositional properties, such as elasticity or flexibility or solvability, actually subsist in the object or whether they may not just be the product of metaphysical speculation, an illusion of knowledge and language, involving a set up that goes strictly beyond the object and the actual fact – what the logical empiricists call *counterfactuals*: "If the object were submitted to a given force, it would bend," when, in actual fact, that is to say, at rest, it is neither submitted to the force nor it bends. This is perhaps the reason why philosophers of language – that the empiricists usually quickly become once they realize they have nothing to say about anything outside language – have assigned in the general realm of meaning, or more specifically, the realm of *interpretation of empirical reality* known as scientific theory, the place of the dispositional terms. So in confronting, like we said, Young's modulus with its meaning, namely, with the theoretical model and the formula yielding the magnitude of the deformation, there might be more than just a play on the word "meaning." We are in fact just sending back the scientific term, and with it the whole concept underpinning it, to the framework that first bestows meaning on it.

Implying volatility from the option price instead of the underlying share price is, to the majority of people, a clever, if a rather bold, artifice

Implied volatility is the only observable number

I, for one, would be very happy to argue that, in the operation of implying volatility from the option market, the concept of implied volatility is actually sent back to its place of meaning, and to insist that there may even be *no meaning* to the concept of volatility – I mean the volatility to use in option pricing, what else? – outside this operation. Not everyone would agree, of course. Not only because volatility, to the majority of people, is not a disposition of the option – something that would be unobservable for the reason solely that you could not observe it directly in the option, but would need to stress and press the option price, against movements in the underlying, against the passage of time, against everything that varies in the Black-Scholes formula, and why not against the prices of other options, in order to infer it – but because volatility, to the majority of minds, is *independently* unobservable. It is a hidden property of the motion *of the underlying share*.

It is believed that this hidden property can be estimated statistically from the time series of the underlying. Provided, that is, you believe you can first represent something as the *random generator* of the underlying market, second, that you certify it even makes sense to speak of *estimating* the moments of that random generator (no essential uncertainty), and finally, that you trust the random generator will be *stationary*. To the majority of people, then, "volatility" means the volatility of the underlying share (not a disposition of the option), something that belongs to the share and is inscribed in its market; something that could have been there for us to read or observe directly, as if from God's point of view, if volatility were not precisely a statistical property, and *for this rea-*

>

son, unobservable.

In other words, implying volatility from the option price instead of the underlying share price is, to the majority of people, a clever, if a rather bold, artifice. "Of course," we say, "it would have been preferable to infer volatility from the observed behavior of the share *then* to go ahead and apply it in the formula for the pricing of the option and for hedging it. Of course, it would have been even better if the right value of volatility was bequeathed to us by some higher authority or some superior observer (God?) and the task only left to us to pick the option we want to trade and rebalance its hedge at the appropriate times." In believing in this utopian world, we are in fact just following the logic of derivation of the option value as it was molded for us originally in the Black-Scholes paper, and rehearsed, ever since, in every single theoretical paper on derivative pricing; a logic that writes the stochastic process of the underlying share at the beginning of the paper, and derives option value at the end.

Even though everyone would agree that the value of volatility is almost always unknown, the lasting impression that the Black-Scholes tradition has left on people's minds is that volatility has at least to be given. (This is the unshakable metaphysical assumption underlying the entirety of derivative pricing.) For how else to even set up the theoretical argument leading to the derivation of option value? How else to even make sense of the fact that the derivative is called "derivative" and has to succeed, in the order of derivation and signification, to something more original and initially given, namely the process of the underlying? Only because volatility is supposed to exist, or be representable in the last resort, regardless of whether it is known or unknown or known to be stochastic, is an article like the one by Ahmad and Wilmott[1] ultimately motivated. "How to delta hedge when your estimate of future actual volatility differs from that of the market as measured by the implied volatility?" ask the authors at the beginning. Although their question seems to carry no metaphysical load and they seem very careful in selecting terms ("estimate", "measured") free of presumption about the actual value, behavior or even existence of volatility, their article, as we will see, soon lapses into just this assumption when it becomes a matter of quantifying the results of hedging. For the moment, let us recognize that the question they pose is, to say the least, perfectly appropriate, for to talk of option pricing and delta hedging means that we are already armed with the Black-Scholes formula and that the only remaining thing is to determine the volatility number that we should insert in it when hedging. To that purpose, we may or may not pay heed to the number that we measure to be "implied volatility." We may or may not have our personal estimate of volatility, but if we do, it will most probability be different from the only publicly available number, namely implied volatility, because "the market," as the authors go on to say, "does not have perfect knowledge about the future."

At this point you feel that the article will start addressing the really hard problem of *option pricing and hedging when the only observable number is implied volatility*. Since the question is that of the delta hedge to use in practice, you think the authors will elaborate decision-making under empirical evidence – when reality does not conform to the model and the market is your only reality. You feel that their article, because it is first to break the *analyticity* of Black-Scholes and to no longer assume that realized volatility, implied volatility, and the volatility to use in the hedging, mean the same except in the Black-Scholes analysis, will tackle at last the *real* meaning of implied volatility and the *real* hedging issue. And by that I mean that implied volatility can only be stochastic – because it is read from the option *market*! – and that hedging cannot be, as the paper later flatly suggests, a hedging whose performance we shall be able to quantify by virtue of a "forecast of actual volatility that *will turn out to be correct*"! I shall argue on the contrary that, once the step towards implied volatility and its real meaning is taken, we can no longer preserve a framework where actual volatility is still thought as "given" nor, equally, maintain the metaphysical picture that enables us later *to note that our forecast of actual volatility "actually" turned out to be correct*. We can no longer accept that "implied volatility" shall rank among the other estimates, with just the distinction and just the appeal that it is objectively measured against the empirical option price when the others are merely subjective.

The new meaning of forecast

I shall argue that the step towards implied volatility is in fact so radical that it ought to change the whole attitude towards statistical inference. Instead of endlessly brooding over the fact that future volatility is very hard to estimate from the history of prices of the underlying and suddenly marveling that this new concept, implied volatility, may just be offering us, at long last, the *forward-looking* estimate we were looking for (save that it might, like any other estimate, not "actually" turn out to be correct); instead of maintaining an old conceptual scheme and welcoming implied volatility as the late – no doubt quite original but nonetheless merely additional – newcomer, I propose to turn the conceptual scheme on its head and argue that it may truly be only now, with the advent of implied volatility, that we are finally making sense of the whole idea of *forecast*.

And by "forecast" I will consequently no longer mean what has all along been understood

I shall argue that the step towards implied volatility is in fact so radical that it ought to change the whole attitude towards statistical inference

in the old conceptual scheme, namely a guess about future volatility of which we could say, as of today, and because volatility is supposed to be *given* today (that is to say, it is observable from God's point of view and at minimum representable from ours), that it will turn out to be correct or incorrect. I will not even stop at that first assault against the notion of forecast, essential uncertainty, which manages only to switch the positive to negative and counter the possibility of a forecast with reservations and arguments no less representational, namely, that volatility shall not be liable to be forecast today for the reason that we may not be sure that it has meaning or that the probability distribution of stock returns admits of finite moments, or more briefly, for the reason that the random generator of stock returns may not be of a known type and that we may not even be able to assign a meaningful probability number to the next random move.

I shall aim higher than essential uncertainty because essential uncertainty, as irrevocable as it may sound, in fact still keeps for the "future" (and for its unfolding through the staging of a time series) the same meaning and the same expectations as does the econometric vein it is criticizing. It differs only in filling up its representation with the *impossibility* of the forecast and the *incommensurability* of the econometrics. (This is what I meant when I said that it was the same conceptual switch, except that it is now set on the "off" position.) What I have in mind, by contrast, is a generalization of the notion of forecast that will surpass both the view that we may have a forecast today and the view that we may never have one. I will propose that a forecast is not just something that we can *have* or not *have* and carry with us as we march into the future, but that it is, generally, anything we could *do* today about the future. (So to trade is essentially to forecast.) Instead of lamenting the fact that we are handed over, with the market, a very strange beast indeed that fits none of our forecasting schemes and makes a joke of our econometric paradigm, and instead of wondering what sense to then make of the concept of "implied volatility" (a concept that seems to want to say something about the future variance of the logarithm of returns – more truthfully so, for that matter, than all the other backward-looking estimates – yet is almost immediately prevented from saying it on pretence that "actual volatility will turn out different" or "that the market doesn't have perfect knowledge of the future" or any other story or metaphor or personification based on words such as "actual volatility," "turn out," "knowledge," "future," etc., which all belong in the old representational schema), I propose that implied volatility be the first answer to the almost incredible task of predicting a future statistical property *from the future* and that the *market* – not the reduced and confined market that we keep modeling as a stochastic process at the beginning of the paper only to realize at the end that it escapes modeling and processability altogether, but the market at large, the market that above all includes derivatives and gives implied volatility its real meaning – I propose that the market be the technology that is precisely handed over to us to give new meaning to the "future" and new meaning to "forecast" and therefore to f*orecast the future*.

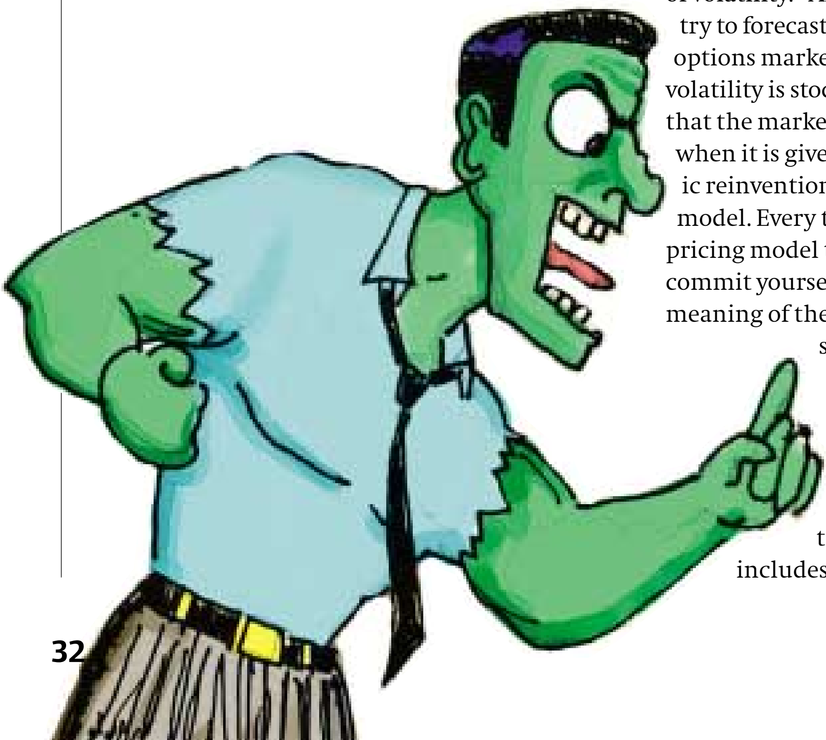
The real meaning of implied volatility and the redefinition of statistical inference

I have argued in a previous column that only with the advent of derivatives does the market acquire its real problem and its real meaning. The old paradigm – let us call it the "econometric paradigm"– which assumed the existence of a random generator and turned the whole problem into the question of how to reliably infer its moments from the time series of prices, was patently exploding its limits and crying out for its change. Along came the derivatives, and for a moment they gave us the hope that something concerning the future could at last be written today and traded today. Unfortunately they were caught in the old paradigm too: the formula that was used to price them off the initial process was simply used in reverse, as if it was meaningful to do so, and the chance was completely missed to understand what the derivatives really had to say. Instead, the volatility of the underlying share was said to be "implied" from the derivative price, and the only sense that the old paradigm could make of that act was to say: "This is the market view of volatility", "This is how the option market forecasts volatility," or even worse: "This number may be different from actual volatility because the market may not have perfect knowledge about the future."

As if the market could *know* anything! As if there were a relation between the derivatives market and that number at the beginning of the paper, which is called the volatility of the underlying share and written in a specific context for the sake of pricing a specific derivative! Implying the value of volatility from the options *market* might be, as a matter of fact, the last thing to do! One usually looks for a firm and stable empirical reality against which to perform a measurement and solve a given inverse problem. For instance, one selects a solid material and exerts tension upon it in order to infer Young's modulus. As changing and fleeting and revisable and unwarranted as elasticity theory may be to the empirically minded, at least the material we are leaning against to invert our equation will not change! If option prices were listed in some immutable register, then it might be a good idea indeed to infer volatility from that unexpected source of information. But we are talking here of inferring volatility from the option *market*! Instead of closing the mystery of the "first" market (that of the underlying share) and the mystery of its volatility, we are now introducing a much bigger mystery right into the heart of our problem and opening a much wider meaning for the word "market" than was initially intended by our model.

I say: Of course "implied volatility" is meaningful! Only it is not the meaning that the old paradigm was able to tolerate. What was wrong in the act of implying volatility from the option price is not the belief that the derivatives market may forecast the future (for surely the derivatives market is *all about* writing and trading the future!) rather, it is the act of inveterately folding back the future into the same old lines of writing that went before. It is the belief that the derivative, by virtue of the formula and the underlying process that were written to derive its value, can then, retroactively, register a value for volatility that will be later compared to the value that actually took place. All of which taking place on the same plane and going back and forth on the

If implied volatility is followed through all its implications we find that it perpetually leads to the devastation of its concept



same line of course, when in fact the act of implying volatility from the option market is already, by itself, contradicting the previous lines of writing! The chance that is offered us by "implied volatility" is that of holding, in the same hand, both the formula to derive the option value along a certain line and the realization that, as a consequence of the term "options market," the resulting implied volatility will be stochastic and will contradict the certain line. It is the chance of holding both the writing and the negation of the writing – that we may then call the "trading."

In my previous column, I call implied volatility the "most original concept." In it, I believe, the market takes its origin. Not the market that has been around for millennia, of course, but the market that has been opened by the derivatives, the market in the newly defined sense of *forecast of the future*.

The market is not interested in forecasting the *future* in the reduced, econometric sense of the term. You try to imply volatility from the options market and you realize that, by doing so, i.e. by the very meaning of implied volatility, you turn the volatility number into a stochastic variable and you end up contradicting the assumption of the Black-Scholes formula. What implied volatility therefore really means is *stochastic implied volatility*. A mnemonic way to put it is to say that "what is implied by implied is volatility of volatility." And the iteration is endless. If you try to forecast the future of volatility from the options market then the real forecast is that volatility is stochastic. Therefore the only future that the market is interested in forecasting, when it is given its real meaning, is the dramatic reinvention and rewriting of the previous model. Every time you calibrate a derivative pricing model to the derivatives market, you commit yourself to recalibration by the very meaning of the word "market," therefore to the stochasticization of the model.

Also when you think about it, the reason why we turned to the options market in the first place, to try and imply the volatility of the underlying, is that it is a market, something that includes a lot of incidents and that settles something big, worthy of consideration, in any case, something we feel is adapted for the job because it, too, is a "market" – just like the market of the underlying whose volatility we are trying to forecast. What I propose is to be radical about this. Why stage the market in two different levels, the market of the underlying and the options market? We can no longer go to the derivatives market – which, by its derivative nature, definitely implies something about the underlying – and at the same time ignore its being a market and deny that options are traded and their prices are stochastic. If it is the same *tradability* that is giving credence to implied volatility on the one hand, and making it stochastic on the other, then I wish to define the market at the hinge exactly! The market as a forecasting technology, and as such inseparable from the derivatives, requires exactly that hinge. It is that hinge. (It is in the "re-" of "recalibration," as I stressed in my previous column.) When all this has been said and our new forecasting technology so defined, the only problem that we are left with is that we had originally gone looking, as conscientious econometricians, for a forward-looking means of statistical inference, and that we ended up with a lot more than what we wanted and perhaps with something different: with nothing short of a complete redefinition of the notion of statistical inference!

The market *fore-casts* the future. It literally thrusts the future forward, beyond the range of possibilities that was meant to it by the model just before and the representation just before. There is literally something amazing about derivatives and that is that when they are summoned to complete our knowledge (by feeding us back with the supposedly unknown volatility number), they precisely open a new level of questioning and a new range of concerns. This is just the reflection of the conflict of meaning between "implied volatility" as a word and "implied volatility" as a concept. Not mentioning that the sole purpose of the activity of writing derivatives and valuing them is to trade them, a principle we can now rephrase as: "The only purpose of framing derivatives in a given representation and model is to unsettle that representation and model." If implied volatility is followed through all its implications we find that it perpetually leads to the devastation of its concept. This is why I suggested at the beginning that implied volatility was not just a word or a concept.

The false recalibration problem

We saw that immersion in the derivatives market and the implications of implied volatility result in the forecast of a much more "serious" future than was initially conceived by the econometric paradigm. This poses the question of derivative hedging all over again, all the more seriously now that theoretical hedging is supposed to work within a given representation of the future behavior of prices (both of the underlying and the derivatives) and that derivatives markets face us with a future of such a nature as to escape any given model and representation. Technically, our problem can be described as *hedging through recalibration*, or hedging against model risk. The reason this problem is particularly hard is that, since hedging requires a model, you will need a model for model risk; how then will you hedge against the risk of *that* model?

There is a variant of the problem that I will call the "false recalibration problem." In the false recalibration problem, you make an assumption about the way your model changes; you just assume a model for model risk. For instance, you assume that everybody is using Black-Scholes with constant volatility, when in actual fact volatility is stochastic, and you try to quantify the profit and loss of their hedging strategies. How this is achieved is by sampling paths for the underlying and its volatility in your favorite stochastic volatility model (that you assume to be reality – for instance, Heston), and by integrating the infinitesimal fluctuations of profit and loss of the option and its delta hedge along these paths. The delta is of course computed using the Black-Scholes formula with constant volatility, with the extra question (which may lead to different results) of whether this volatility should be kept constant or updated along the path.

An even simpler version of the false recalibration problem is the study provided by Ahmad and Wilmott. Here the authors fantasize that actual volatility will, in effect, be constant (therefore they will only sample paths for the underlying), and they study the effects of using the Black-Scholes formula with a different volatility number, either for pricing the option or for both pricing it and hedging it. The Black-Scholes formula is still very widely used by option traders and seldom does actual volatility turn out to be the same as the one they used in the formula. For this reason, I believe the study of Ahmad and Wilmott is very useful and very interesting. However, I wish to criticize it from precisely the angle of the question it poses at the beginning, not the results it produces at the end: "How to delta hedge when your estimate of future actual volatility differs from that of the market as measured by the implied volatility?" I will ask: "Does their study help us selecting the hedge *a priori*?"

Indeed, what troubles me – and this is why my criticism will reach, beyond Ahmad and Wilmott, to all the other versions of the false recalibration problem – is that they need, like all the others, a model for model risk. And the model proposed by Ahmad and Wilmott is that volatility shall be constant (instead of stochastic, as in the first example), only different from what is used for hedging. (They could have modeled volatility as stochastic of course, but this would have only made their computation more complex and my criticism would hold all the same.) What troubles me is the following sentence in their article: "Imagine that we have a forecast for volatility over the remaining life of an option; this volatility is forecast to be constant, and *further assume that our forecast turns out to be correct.* " Not that I mind that we should have a forecast for volatility and, moreover, that we should forecast volatility to be constant or to follow some given stochastic process. This part of their sentence belongs to the subjective domain, hence is unattackable. On the contrary, when all we face is uncertainty, our decision-making process has no choice but to lean on forecasts and prior beliefs. It is the rest of sentence that I do mind, for it makes it all sound as if the problem has all of sudden become one of *back-testing*: "Volatility was observed to be constant and equal to σ over a certain period of time, (Alternatively: "Volatility was observed to follow a certain path over a certain period of time"), and the option trader was witnessed to buy and hedge the option with a certain volatility $\tilde{\sigma}$, can you please compute his profit and loss?" We do so *a posteriori* and this tells us nothing about the delta to use *a priori*. However, if you insist that Ahmad and Wilmott are not back-trading, that they are here with us today, only they are projecting a *fiction* ("Assume that our forecast will turn out to be correct") such as all theoretical models in fact are, then I will argue that there is no more to their paper than a couple of things: a) a new proof of the Black-Scholes formula and b) a variation of the Ito expansion that normally leads to the Black-Scholes formula using, this time, a different delta than the one prescribed by the formula.

To my mind there can only be two points of view, and when you embrace one of them you must do so totally and exclusively: the point of view of the model and the point of view of its critique, or, if you will, the object-level and the meta-level. I am not disputing the quantitative value of the work by Ahmad and Wilmott or the reach of their results. All I am saying is that their article is unsettled between the two points of view, and this produces a tension as to what it intends to achieve. So let me try to recount their paper, adopting alternatively each of the two points of view exclusively of the other.

• *From the point of view of the model, or at object-level*: Assume the volatility of the underlying share is constant and equal to σ; compute the cost of the dynamic hedging strategy using, in one instance, the delta prescribed by the Black-Scholes theory under volatility σ, and in the other, a different ratio obtained by a different algorithm. In the first instance, you re-demonstrate Black-Scholes and your net profit and loss is the difference between the Black-Scholes value of the option under σ and whatever price you paid for it at the inception of the trade. In the second instance, it will all depend on the algorithm you used to compute the hedge. Ahmad and Wilmott then show some interesting results, such as the path-dependent character of your net profit and loss when your delta is computed with the Black-Scholes formula using a different number $\tilde{\sigma}$ or the guarantee that the P&L of a long option position will remain positive so long as σ is greater than $\tilde{\sigma}$. When it is further assumed that the option trades daily in a schmarket and that that number, $\tilde{\sigma}$, is its schmarket implied volatility, Ahmad and Wilmott remark that the marked-to-schmarket underlying stock fluctuations will cancel the marked-to-schmarket option price fluctuations, which is just the definition of hedging under schmarket implied volatility. Eliminate the word "schmarket" from the previous and see that the sentences where it occurs will detract nothing from the fact that the article of Ahmad and Wilmott is, from the present point of view, just an exercise in stochastic calculus.

• *From the point of view of the critique of the model, or at meta-level*: This is the point of view where we become reflective and where the terms, which I have tried very hard to keep as uninterpreted as possible in the previous paragraph, start acquiring a meaning and an interpretation. Here it is asked what "implied volatility" really means, what the "market" really means, and what it means to face an option market where implied volatility is the only observable. Also it is asked what the future really means and what delta we should use. This is basically the point of view that I have been developing all along in the present article. Now what is true, from this point of view, is that option traders *will* use implied volatility to determine the option delta. As a matter of fact, *that is* its only use since the option price is tautologically given. And I will stop here in my recounting of the article by Ahmad and Wilmott from the critical point of view. Or else I will engage into investigating the real meaning of implied volatility (which, as we saw, is stochastic implied volatility), into solving the problem of hedging through recalibration, etc. Under no circumstance will I step forward into the "false" future and write: "Imagine that we have a forecast σ for volatility and further assume that our forecast turns out to be correct," for that's only the prelude to writing the nice stochastic integrals of the previous point of view, and that is forbidden in the present, radical point of view.

Now we can understand why Ahmad and Wilmott had to resort to the rather circuitous formulation: "Imagine that we have a forecast for volatility over the remaining life of an option; this volatility is forecast to be constant, and further assume that our forecast turns out to be correct," instead of just saying: "Assume the volatility of the underlying share is constant and equal to σ." Indeed their paper has a critical and reflective pretension and they cannot just accept to collapse it in the first point of view. They live in a world where the Black-Scholes formula exists and has already been programmed in the trader's calculator (as a matter of fact they wonder what volatility to use in that calculator), whereas the object-level is where the formula is derived.

A model which would in a sense incorporate its own meta-model, in an endless chain of upgradings

So they have no choice but to speak first of a "forecast" and of "implied volatility," as if they were on the brink of giving these words the full meanings that we did, then to collapse in the "false" future which is in fact just a rehearsal of the past.

The true recalibration problem and the new logic

It all leaves us with our first question: "How to hedge, and consequently price, an option, when the only observable number is implied volatility?" In other words, we are left with the hard problem of derivative pricing and hedging when we know that derivative trading really implies recalibration. The problem is so whole and the break with the old paradigm is so pure that I will call, at this juncture, for the emergence of a new framework, perhaps even a whole new logic, for option trading and hedging.

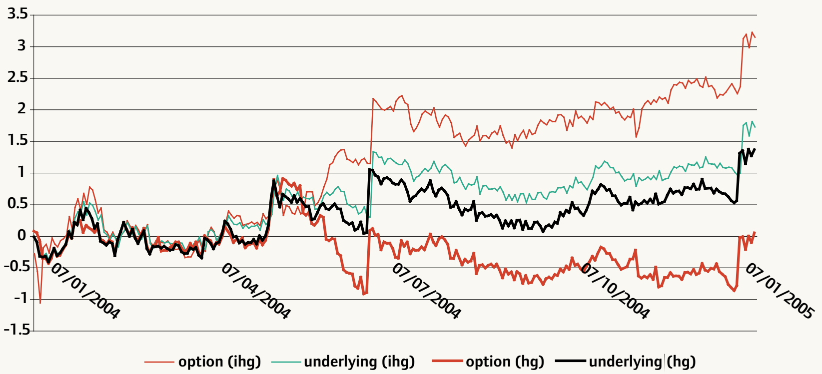
From all the critique that went before I shall extract as my first rule that "Every time you imply a volatility parameter from the market (and "volatility parameter" is here more general than Black-Scholes volatility: it will mean, generally, the parameter of any advanced smile model that we may encounter in the build-up), this will be for the purpose of determining some hedging ratio." Typically, you compute implied volatility in Black-Scholes in order to compute the corresponding delta hedge.

I shall enunciate as my second rule that "Every time you do the previous, you must prepare your model for the next level – the level where it is acknowledged that the implied parameter will therefore be stochastic." This is the recalibration step where, typically, by determining as your strategy that you shall be implying volatility everyday from the options market, you will be committing yourself to stochastic volatility.

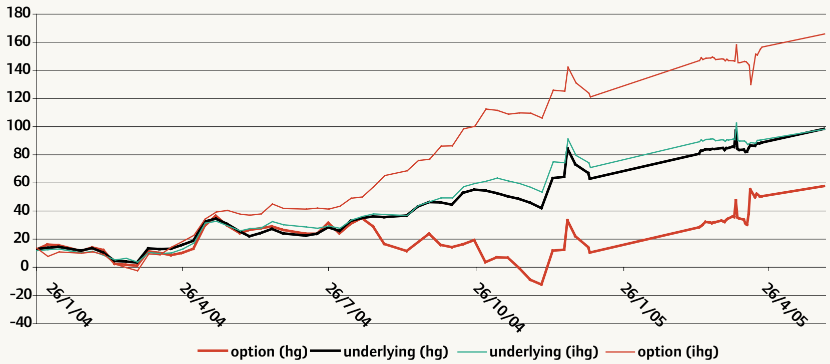
"Preparing your model" will then mean that your model was, already from the day before, prepared to take in stochastic volatility and to be calibrated against the simultaneous market prices of at least two options, or option smile. This is just saying that your model was a smile model all along and that you were calibrating it to the option smile.

But then, by application of rule number one, this will mean that the model parameters that you are implying at this stage – typically the parameters of the stochastic volatility model – will be used to compute a corresponding hedging ratio (in this case, against stochastic volatility). Obviously, the second option you are calibrating against, whose price differential relative to the first one is indicative of the volatility smile, therefore of stochastic volatility, will be the ideal candidate as a hedging instrument against stochastic volatility. So the second hedging ratio you are computing should apply to that option.

This, by the way, suggests that I enunciate as my third rule that "Every time you observe a deviation in the option prices relative to what the current model predicts, you must interpret that as a signal to upgrade your model to the next stochastic level, and you must pick the option that deviated as a hedging instrument against the next stochastic factor." Typically the out-of-the-money put deviates from the flat volatility assumption; you upgrade to a stochastic volatili-



GM Comparison of the cumulative P&L of the hedged GM convertible, from January 2004 to January 2005, using either the underlying or a combination of the underlying and one option as dynamic hedging strategy, in a constant volatility model (IHG) and a stochastic volatility model (HG).



Tyco Comparison of the cumulative P&L of the hedged Tyco convertible, from January 2004 to March 2005, using either the underlying or a combination of the underlying and one option as dynamic hedging strategy, in a constant volatility model (IHG) and a stochastic volatility model (HG).

ty model which you then calibrate to the implied volatility skew, and you prepare yourself to selecting the put as hedging instrument. (Incidentally, this tells you how utterly mistaken the approach is, where the observed deviation is folded back into the current model instead of triggering an upgrading to the next. For instance, those who try to explain option volatility skew or term-structure by making volatility a function of underlying stock price and/or time instead of making it independently stochastic, are in effect reducing the number of hedging instruments when everything seems, on the contrary, to be pointing to the necessity of increasing it. They hang desperately to a picture where underlying stock and time are the only state variables, and underlying stock the sole hedging instrument.)

Reality at last

Our list of rules seems to delineate quite an impossible model. A model which would in a sense incorporate its own meta-model, in an endless chain of upgradings that would be determined, at each stage, by the variety of derivative instruments we are calibrating against, and the variety of their market prices. Such indeed seems to be the answer to the question of hedging in the face of the true future, that is to say, in the face of recalibration.

I have suggested, in a recent publication [2], that the regime-switching model might just offer the possibility of such an "impossible model." There, I elaborated the idea that the regimes are variables with no names and no pre-determined level of "stochasticity," and that freedom was therefore left completely to the procedure of calibration and recalibration, in other words, to the future, to determine the level at which the model would operate. When the option vanilla surface was found insufficient for determining the smile dynamics, calibration was attempted against path-dependent options, such as barriers and cliquets, which are sensitive to the future smile. And when they, in turn, are found insufficient, calibration will be attempted against more complex structures still. Every instrument used in calibration is liable to be used in hedging. Needless to say, such an open model can only be tested against real market conditions. This was attempted by my co-workers Pedro Ferreira, Philippe Henrotte and Willy Lorange, in a study that they will soon publish separately. I will close the present article by showing the graphs (taken from their study) of the cumulative profit and loss of a dynamically hedged position spanning a period of one year, when the model is recalibrated everyday to the market prices of various derivative instruments, and the corresponding hedges executed at the corresponding market prices.

More specifically, the model is the regime-switching model of volatility and hazard rate discussed in [2]. It is recalibrated everyday to the full implied volatility surface of vanilla options written, in one case, on GM and, in the other, on Tyco and to their full term-structures of CDS market spreads. The hedged portfolio consists, in both cases, of the convertible bond issued by the underlying company and a choice of hedging instruments, either the underlying alone, or the underlying and one option. Two models are also compared. The first (IHG) assumes volatility is constant only implies it everyday from the (actually stochastic) option prices. In this case the option is only used to hedge the jump to default, as the model does not incorporate the assumption of stochastic volatility. The second (HG) incorporates that assumption; therefore the option is used to hedge both the jump to default and stochastic volatility. You will notice that the P&L, using the stochastic volatility upgrading HG with the underlying and option as hedge, is more durably centered around zero than in the other strategies, for both GM and Tyco.

REFERENCES

[1] Riaz Ahmad and Paul Wilmott, Which free lunch would you like today, sir?: Delta hedging, volatility arbitrage and optimal portfolios, *Wilmott* magazine, November 2005.

[2] Elie Ayache, The equity-to-credit problem (or the story of calibration, co-calibration and re-calibration), *The Best of Wilmott 1*, pp 79–115, Wiley, 2005.