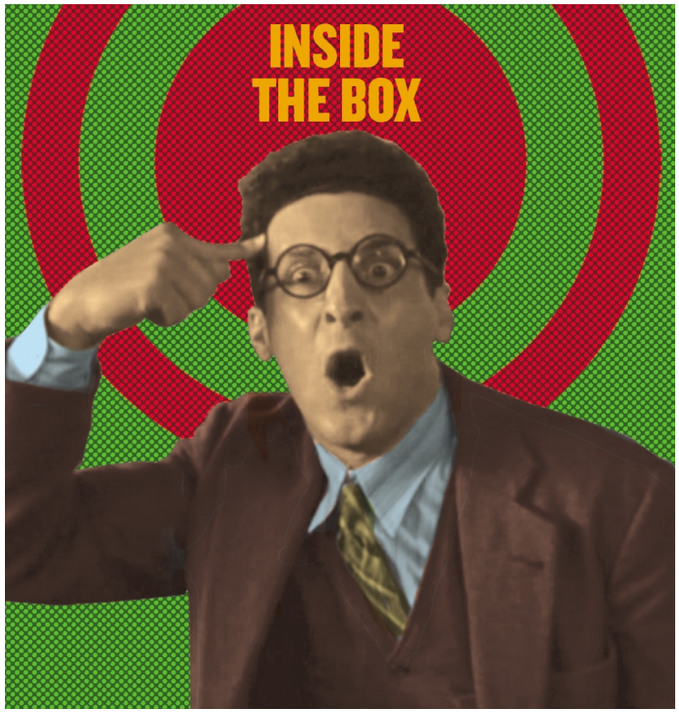
I Am a Creator!*

Elie Ayache explains why you must always think outside the box, yet keep the box with you at all times



Why we have always used the Black–Scholes–Merton option pricing formula

One should always look at option pricing from a market-maker's point of view. There is a paradox in market-making which can be summarized in saying that a good market-maker has got both to make, i.e., create, the market and be-in-the market, i.e., adapt himself to it and somehow "follow" it. So who is the creator here, the trader or the market? Who makes who? The market-maker has got to be both an "original author" and a copier. (My favorite example is Pierre Ménard.) Only this way can he consistently make money. This is what his work is all about.

It is only fair that you make money if you do your work. As market-maker, you shouldn't be "guessing" or predicting the market. This isn't how you produce original work in the market. Immersion, or work, in the market, has nothing to do with probability whatsoever. When someone like Nassim Nicholas Taleb says that traders who make money are just "lucky fools,"[1] he is typically trapped in the "probability" view of the market. He thinks of random generators and probability distributions instead of thinking of writing and market-making.

The breakthrough brought about by Black, Scholes, and Merton, when rightly interpreted, can help us make sense of the paradox of the market-maker. To my mind, the breakthrough lies in the *dynamic replication* of derivatives. (So strictly speaking, it is to Merton that we owe it).

What's really important in *dynamic replication* is the word "dynamic." You shouldn't really care whether the replication is perfect or not. BSM created dynamic traders. It doesn't matter whether dynamic continuous (or not) replication works perfectly or not in practice; it doesn't matter whether your yearly P&L is really kept under perfect control by dynamic replication. Market-makers keep shuffling their positions anyway, and they tend to offset the risk of options using other options and not just the underlying. What matters is that the directive of dynamic replication keeps you "rooted to the spot" so to speak. That it keeps you pinned to the floor. Whenever you buy or sell an option as a market-maker, you have to immediately delta-hedge it, just so you can address the next broker asking you for a quote. You rarely see dynamic market-makers trading options in one or two cents worth of bid-and-ask spread, and not rushing to execute their delta-hedge at once, the moment they are hit.

It doesn't matter whether market-makers are using BSM (with adjusted, or fudged, or "smiled" implied volatility, of course) or some other evolved jump-diffusion stochastic-volatility model in order to dynamically hedge. Good for them if they are using something better than BSM. It doesn't matter. The point is: *they are dynamically inserted in the market*. They trade it tick by tick, or at least they follow it tick by tick. They are the denizens of the minute, microscopic, stochastic path. They quote an option, they trade an option, and they hedge it at once. Just so they can move to the next trade.

Options pits started being crowded with market-makers thanks to BSM. Those people had the impression (again, it doesn't matter whether it is a false impression) that they could *make the market* in the option because they had a formula or algorithm that allowed them to make quotes in options out of quotes in the underlying.

The transformation that was to take place in the market-maker's "organism" can be restaged as follows:

1. Starting from the price process of the underlying, BSM enabled the market-maker to get involved in the *pricing process* of the option.

2. This pricing process meant both quoting an option price and executing the delta-hedge.

3. Through the dynamic delta-hedging and the anxiety that it generates (Will I execute it

* Quote from the movie *Barton Fink* (1991), directed by Joel Coen, starring John Turturro and John Goodman.

right? When to rebalance it, etc.), the market-maker penetrated the market. He penetrated its volatility and he could now feel it in his guts. In a word, he became a *dynamic trader*. He now understood – not conceptually, but through his senses, through his body – the inexorability of time decay, the pains and joys of convexity.

4. Only because he assimilated volatility in this way (in his own body) was he now able to *invert* the model against the market price of the option and to compute *implied volatility.*

5. This is the crucial step. And yes, the claim here is that *implied volatility can only have meaning for dynamic traders*, that is, for traders who perform dynamic replication of the option.

6. Only because he was able to compute implied volatility was the market-maker able to discover the *implied volatility smile*.

7. The implied volatility smile is the reality of options markets, not BSM (so I agree here with all the critics of BSM); however, dynamic replication is necessary in order to face the implied volatility smile.

8. The implied volatility smile signals the *return of the market*. It is original work produced by the market, not by the market-maker. The market-maker can only get dictated the volatility smile.

9. From the pricing process of the option (which was still of his making), the market-maker moved, with the volatility smile, to the *price process* of the option, now understood as an independent price process.

10. With his dynamic replication formula, the market-maker sits at *the hinge between the two price processes of underlying and option*. He both makes the market and he is dictated the market. He can both be an original author yet be-in-the-market.

The volatility smile is what both fulfils the meaning and purpose of dynamic replication and gives back its originality to the market. The volatility smile is the ultimate truth. It is a mistake to think that the implied volatility smile calls for a superior model that would explain it. When the sociologists of finance (Donald MacKenzie) observe that the options markets have enacted the BSM model during the period from 1973 to 1987 and when they subsequently ask what different model options markets have been enacting since 1987, they make this mistake.[2] Espen Haug and Nassim Taleb make the same mistake when they argue that traders who fudge volatility in BSM *are not in fact* using BSM.[3] As Pablo Triana notes: "Taleb and Haug explain that when traders fudge the volatility, they are actually (unbeknownst to them) using a completely different and superior model, one that was developed by different authors long before BSM."[4]

Although MacKenzie and Triana seem to disagree on the direction of the progress of science (as the first seems to be looking for a model that would succeed to BSM when the second seems to recognize this "superior model" among the antiques that were developed long before BSM), they are both essentially asking the same question, namely: *What model (different from BSM) are option traders enacting?* They both expect, as an answer, a representable and identifiable model occurring on the same reference plane as BSM. As if some superior being could open a box one day and say to the traders: "I have a gift for you. I now give you the model that you've been using all this time without knowing."

It seems to me, on the contrary, that what model the traders are using can only be the model that they are *literally using* (and that companies, like the one I run, ship to them). If it is BSM, then it is BSM; if it is something else, then it is something else. As for the question of what is really going on and what *true model* can account for the entire phenomenon, as if surveyed from God's point of view, I don't think the answer can belong on the same plane. The "model" that would explain the traders' *usage* of some option pricing model in some option market is indeed a *superior model*; however, it is not a mathematical model and it is not superior in this sense. You must understand its "superiority" in the sense that the model shall belong to the superior plane of the critique of the science, or philosophy. Technology alone, and no mathematical model, can provide the answer.

I am not saying we shouldn't use a better mathematical model than BSM, and as a matter of fact, I am big fan of jump-diffusion stochastic-volatility models and dynamic hedging in incomplete markets; however, this better model will also generate its own "volatility smile." Enough to calibrate it and recalibrate it every day to the options traded prices for its parameters to become stochastic in their turn, thus breaching its theoretical assumptions and generating a "smile" of higher order.

The truth is in the volatility smile of whatever order it may be (not in some ultimate model). This is what the dynamic market-making of options entails. I don't think the truth is in regressing to models belonging to the predynamic replication era. I don't see how dynamic market-makers could handle those models. Definitely, Taleb's best book on this subject is *Dynamic Hedging*.[5]

Dynamic hedging in models superior to BSM involves trading options against options, mind you. When jumps and stochastic volatility become an integral part of your dynamics, hedging à la BSM, using only the underlying is no longer sufficient of course. However, the argument remains that what is most important is dynamic replication. For, only this way can the market-maker really belong in the market, in the midst of the price processes.

The lesson we should retain, I think, is not the lesson of BSM or dynamic hedging per se. The full lesson is BSM + implied volatility smile, and it can only make sense through dynamic replication. Its other name is recalibration.

Recalibration is the ultimate process. *It is the other name of market-making*. Use any model you want: you will always need to calibrate it to market in order to compute a (perhaps multiple) dynamic hedge, and this means you will recalibrate it. The only reason why BSM is still widely used is that it is used in reverse. (From here, draw the conclusion that traders do not *in fact* use BSM, if that makes you feel better.) It is used in reverse because market-makers use it to compute hedges.

The recalibration process doesn't take place in probability or possibility. You cannot model it by a stochastic process, for if you tried, *that* model would have to be recalibrated in its turn. Yet recalibration is the only reality. It is the reality of the market. Market-making, now that the BSM technology has helped us to decompose it in the sequence above, boils down to recalibration.

There is, in the philosophy of Gilles Deleuze, a characterization of that kind of "limiting process" which is real, yet is not actual (for then, it is too late when it becomes actual), yet doesn't take place in possibility. Deleuze calls it the *virtual*.[6]

It is about time we saw that the market entails a complete overturning of the notion of possibility and probability. Only if the market is extricated from possibility and probability and from the whole category of prediction will it regain its reality and originality, and will the trader deserve to make money without being called a "lucky fool."

The market is real. Any philosophy whose ultimate logical consequence is that people involved in the markets, using market-making tools, are fools, therefore shouldn't be doing what they do, is bound to be wrong.

But, do we really need dynamic hedging in order to get risk-neutral pricing?

There has been much debate, recently, on whether Black, Scholes, and Merton are the true creators of risk-neutral option pricing. Haug and Taleb argue they're not. According to Haug and Taleb, the original contribution of Black, Scholes, and Merton is limited to *dynamic hedging*. As such, it has to be dismissed because dynamic hedging is infeasible in practice. (Haug and Taleb call it "a thought experiment.") It relies on continuous rebalancing of the hedge and no transactions costs, not mentioning the thin-tail property of the Gaussian distribution.

Instead of dynamic hedging, Haug and Taleb propose that we rely on the more robust and indisputable *static hedging*. "The Black–Scholes–Merton's claim to fame," Haug and Taleb write, "is removing the necessity of a risk-based drift from the underlying security, to make the trade 'risk-neutral.' But one does not need dynamic hedging for that: simple put-call parity [essentially a static hedging argument] can suffice."

Put-call parity was well-known in the early days of option pricing literature. Haug and Taleb report that it was extensively investigated and commented, as early as the beginning of the twentieth century: Higgins 1902, Nelson 1904. Consequently, option pricing formulas very similar to BSM were also available before 1973: Bachelier 1900, Sprenkle 1962, Boness 1964, Thorp 1969. Probably the superior models Triana was referring to.

As for trading practice, Haug and Taleb remind us that the knowledgeable option trader just hedges options with options, and the less dynamic the hedge, the better. "People would not trade options unless they are in the business of trading options," the two authors note penetratingly, "in which case they would need to have a book with offsetting trades. For without offsetting trades, we doubt traders would be able to produce a position beyond a minimum (and negligible) size, as dynamic hedging is not possible."

I have nothing against setting the record straight and arguing that Black, Scholes, or Merton are not the true owners of their Nobel-winning formula, because others have derived similar formulas before, or because traders (e.g., Thorp) were practically using this formula before it got published. History of science can be enlightening, especially when it turns into a philosophy of science. But if it doesn't and remains caught in debates of ownership, its runs the risk of becoming itself assimilated to a "backward-looking model" (to borrow Taleb's favorite characterization of BSM[7]).

On the other hand, I am a very strong believer in the benefits of dynamic replication, and I cannot allow arguments, urging a return to static replication, to stand. So how is risk-neutral pricing accessible without dynamic hedging at all? This takes us back to a paper by Derman and Taleb[8] that Haug and Taleb (2008) cite and call "a rediscovery of work by Bronzin (1908)."

It is worthwhile to rehearse Derman and Taleb's argument.

The authors first recall "the plausible and time-honoured *actuarial* way" of estimating the value of calls and puts. Assuming Brownian motion, this leads them to option value as discounted expectation of payoff, where the expectation is taken under real probability, in which the drift of the underlying is μ and its volatility σ. Actuaries are experts in statistics and in the calculations of insurance risks and premiums, according to the dictionary. No wonder the risk premium of the underlying is what first appears in Derman and Taleb's formula.

Second, the authors invoke a static replication argument, or put-call parity, or model-independent arbitrage, to conclude that the underlying drift rate μ and the option premium discounting rate appearing in the option valuation formula should both be equal to r, the riskless interest rate.

Conclusion: We don't need dynamic replication; static replication is sufficient to derive BSM. Using the riskless rate in the BSM formula is not a consequence of perfect continuous dynamic hedging, etc., etc.

Maybe so, but all this arguing makes it look as if the breakthrough of BSM was only the discovery of risk-neutral pricing! In reality, *risk-neutral pricing has nothing to do with hedging* (dynamic or not), hence nothing to do with BSM, and is a much more general condition of modern pricing theory. BSM may have been the first, historically, to throw risk-neutral pricing into light, however, when we, commentators, reconstruct the significance and originality of BSM we have to abstract away from history.

Because a stock has a rather large drift (a fact of life) you end up on average always making money if you buy the stock today and sell it later at some given maturity. Does this mean that the underlying is not well priced today because it is too cheap? Would you call this strategy an arbitrage? Surely not: you earn a return on the stock, also called a risk premium, because this investment is risky. Yet the price process is a martingale, and the spot price is the expectation of the future price discounted by the risk-less interest rate. Of course nothing here hinges on the ability to hedge.

In the absence of arbitrage, a pricing system must be a positive and linear operator. Because the price of the bond is one (assuming zero interest rate), this positive operator can be understood as being an expectation operator applied to the payoff at maturity. If the interest rate is nonzero, you end up with a discounted expectation. What can this expectation be? Under which probability is it computed? Nobody knows, and absence of arbitrage alone will not tell you in general. You know one thing however: *this operator should price the underlying itself correctly*. Pricing the underlying correctly means that the price today is the dis-

counted (at the risk-free rate) expectation of the price at maturity, for our unknown probability. This proves that, for this mysterious probability, there is no risk premium for the underlying, or that its drift is r. That is precisely why any such probability is called a risk neutral probability.

To repeat, risk-neutral probability is another word for "a linear and positive pricing system." *All we need here is the knowledge of the current price of the underlying; no hedging argument is needed.*

Surely, you may ask, are there many possible risk-neutral probabilities? In general the answer is yes. Before going any further, remark that they all yield a drift of r for the underlying (and, by the way, for any other security).

Enter the dynamic hedge.

If markets are complete, every payoff may be perfectly replicated, and in absence of arbitrage, the price of a derivative must be equal to the initial value of the hedging strategy. You end up with a unique pricing system, or a unique risk-neutral probability, which *also* yields the value of the perfect hedge. This is what happens in continuous time for the BSM setting or in discrete time with the binomial tree. In a more general case with incomplete markets, if the security cannot be hedged perfectly, then many pricing systems may be consistent with absence of arbitrage.

Bottom line: the drift is r in BSM not because of perfect hedging but because all pricing systems (consistent with no arbitrage) imply a drift r and there happens to be only one such system in BSM (which is the reason why everybody has the wrong impression that risk-neutral pricing is the consequence of dynamic hedging).

What have Haug and Taleb (and Derman) been missing, then?

So Derman and Taleb's argument is just the rehearsal of the well-known fact that a pricing system should price the underlying and the risk-less bond correctly and be arbitrage-free. And the reason why the option pricing formula they produce (formally identical to BSM yet without the dynamic hedging argument) is not *in fact* equivalent to BSM is that *nothing would then guarantee that the volatility number entering in their formula is the same as in BSM.*

In other words, my contention now is that only dynamic hedging (*and dynamic hedging is what's specific to BSM*) can establish a link between the volatility of the underlying and the volatility number to feed in the formula. Derman and Taleb's reference to the "time-honoured actuarial way" of pricing options is in fact a legerdemain. What can the exact provenance of the volatility number entering in this actuarial formula be? What market-maker, alive in a market and riding on top of an option book he keeps shuffling, can afford to sit and wait until he breaks even in the long run? What long run? Markets trade in ticks.

Only dynamic hedging can establish a link between the volatility of the underlying and the volatility number to feed in the formula

So here is my question:

Assuming that the volatility of the underlying is σ and that *no dynamic hedging is permitted*, what "arbitrage," or generally what "physical forces," could possibly bring someone to use σ as *only viable number* in their option pricing formula? I think this is the question worth asking, as everyone agrees that what's specific to BSM is dynamic hedging and what's in BSM is the volatility of the underlying and not just any number.

Surely, Derman and Taleb would reply that this is part of the actuary's job: surely σ is the historical statistical volatility of the underlying. (Sounds backward-looking?) Sure, why not. But my point is that, absent dynamic hedging, the volatility number you're using in the option valuation formula becomes disconnected from the real, instant volatility of the underlying.

I grant you that "volatility" is a suspicious concept to anyone not trusting Gaussian distributions, but we aren't on that level of criticism just yet. For now, the point is that if dynamic hedging is disabled – say you cannot trade the underlying for one reason or another – then *any* volatility number you care to input in the BSM formula will yield option prices that are arbitrage-free!

So here is the irony: if dynamic hedging were not allowed, i.e., if *volatility arbitrage* were not allowed (and by this I mean arbitraging the option premium against the "real volatility" of the underlying, whatever that means: surely this notion can be extended to non-Gaussian frameworks and to jumps[9]), you could quote options prices using BSM with any volatility number you like (making sure you use the risk-less rate as growth and discount factor) and no one would be able to buy from you and sell to you options, on your prices, and make money against you for certain. You could *use* BSM, despite Haug and Taleb's best intentions. No one could demonstrably convince you not to use BSM, even less so show you "why you have never used it."

Your quotes may be inconsistent with the market volatility smile of course (hey, you are using BSM with flat volatility!) and people may then arbitrage you against the *option market*. But wait a second! Our argument is taking place at a stage where there is no option market yet. We are precisely talking about making the market in options.

As a matter of fact, if only static hedging were allowed, or in other words, if we restricted the notion of arbitrage only to *intrinsic-arbitrage* relations and constraints, such as put-call parity, that hold between derivatives independently of the dynamic model, that is to say, independently of the particular states of the world you are postulating and of the particular probability distribution you are assigning them, then not just families of option prices generated with BSM (using any volatility number) will come out arbitrage-free, but families of options prices generated by any smile model with a pricing kernel.

What of dynamic replication?

The whole purpose of dynamic replication is to narrow the choice of such families of option prices. Yes, dynamic replication does introduce a notion of arbitrage that is no longer intrinsic, i.e., that must now depend on the dynamics you are postulating. From here, it becomes a personal choice whether you wish to step forward as option market-marker, i.e., narrow your choice of families of prices, just in order to expand the scope of instruments you are pricing, or stick with static positions involving only put-call parity or broad model-independent constraints between options prices such as indicated in Merton's "Rational Option Pricing" (1973). How far would you go, pricing exotics only with static replication arguments? I know this to be a quant's favorite exercise (static replication of barrier options, of variance swaps, etc.). But you will often find a hidden assumption concerning the dynamics for that to work (it is a diffusion, or the paths are symmetric, or what not).

I used to like Haug's and Taleb's arguments better when they were defenders of Black–Scholes–Merton

Since I view dynamic replication as just a way of narrowing the choice of option pricing families, it should be obvious that I only care to consider dynamic replication in *incomplete markets*. Of course the assumptions underlying BSM are unrealistic! Of course continuous hedging is a fantasy, not mentioning the Gaussian distribution! I fully agree with Taleb and Haug here. *However, their argument becomes a peril for the technology of derivative pricing as a whole when they start confusing arguments against BSM with arguments against dynamic replication*. As a matter of fact, Fisher Black and Myron Scholes did not themselves believe in dynamic replication! They have long resisted this argument from Robert Merton (see Donald Mackenzie's invaluable account of all this in his book *An Engine, Not A Camera*).

I used to like Haug's and Taleb's arguments better when they were defenders of BSM. "No experienced trader would willingly trade Black-Scholes for another pricing tool," writes Taleb in *Dynamic Hedging*. "An option trader knowing the ins and outs of the Black–Scholes–Merton formula can beat a trader using a state-of-the-art stochastic volatility model," writes Haug in "Know Your Weapon."[10] Now surely, Haug and Taleb were not defending the *assumptions* underlying BSM back then, but the practical usage of the formula, what I call the *technology*. Pablo Triana summarizes the point nicely when he writes: "No other derivatives pricing tool allows practitioners to get agreeable outputs so comfortably. Traders know the model is wrong (not just the issue of a normal distribution, but also invalid assumptions about key issues such as market jumps and liquidity), but they all use it because it is so easy to trick it into generating realistic and satisfactory results. They don't trust Black-Scholes, but they like it."[11] So if Haug and Taleb now come down on BSM, screaming against its assumptions, this is perfectly OK, but then they owe us an elaboration of the technology that they see as candidate to replacing what used to be the good side of BSM and made all the traders like it.

We all agree that options traders do not *in fact* use BSM and that option prices are not in fact "set by some funky mathematical sorcery" (as Triana remarks) and that "the hurly-burly of the trading floor has more to do with it."[12] But the pressing question (pressing from the point of view of the technology, that is) that I believe Haug and Taleb are unable to answer is: "What model *should* option traders be using, then?" The problem is that whatever the model they might end up using (a model their favorite analytics provider would ship to them, for instance), the very logic of market-making will make it so that they would end up, sooner or later, not using that model *in fact* but a different one still. This is so because the usage of any model, however superior it is, will generate the corresponding volatility smile, as I have indicated above.

Cleary, there is an equivocation on the word "use" at the heart of Haug and Taleb's charge against BSM. When I, as an option analytics provider, use that word, I really mean the concrete model that I ship to our customers and that they use literally. Believe it or not, not only the majority of options traders I have interviewed (in the hope of selling them a superior model) use BSM, but so do the majority of convertible bond traders! By contrast, when Haug and Taleb (and now Triana, following in their footsteps) wonder: "Do traders use Black-Scholes?" they pose the question from above, from God's point of view: "What model are traders actually using without even knowing?"

One of the implications of Haug and Taleb's "potentially revolutionary paper" that Triana notes is that "implied volatility, a ubiquitous element of the markets, ceases to make sense" when traders are found not to be using BSM. Implied volatility "simply can't exist," he writes, "in a world where options prices are determined by supply and demand"[13] (and not by BSM).

But, dear Pablo, it is precisely in BSM that implied volatility doesn't make sense! In the strict BSM framework and formalism, there is no such thing as a market option price against which to invert the formula and imply volatility. There is even no meaning to the act of inverting the formula. On the contrary, implied volatility is the very hinge between the two planes I was alluding to earlier: 1) the inferior plane where no mathematical model shall ever answer the question of the model option traders are enacting and 2) the superior plane where the completed technology[14] can alone provide the answer, not to that question, but to the question of the model traders *should* be using. And I don't mean it in the normative sense of the word. What I mean is: What model should traders use (and should we ship to them responsibly), now that we know, by the problem of recalibration, that there is no norm and that there could never be one? (Haug and Taleb, and now Triana, must have despaired of this question ever finding an answer.)

As a matter of fact, if you really analyze the concept of implied volatility and follow through

its implications, like I did once,[15] this in itself is sufficient to outline the shape of the next technology we all need. I have been spelling out the requirements (both philosophical and financial-theoretic) of this technology for the last two years of publications in this magazine. As early as May 2004,[16] I have indicated what the right way of criticizing, and surpassing, BSM should be. If Haug and Taleb are so keen on readjusting the history of the science, it seems they have a lot of catching up to do in terms of the history of the philosophy of the science.

To best address the question of what model, or technology, option traders *should* use, one is advised to pose it from the point of view of an analytics provider to that community. I recognize this is a very difficult question to try to address outside the BSM framework. Why? Because, by the very meaning of a *technological process* – and it is one such technological process that the customer expects from you – option prices that your technology produces must result from a *pricing process*. To the best of my knowledge, when all you have for starters is the *price process* of the underlying, the only way to process that into option prices is through dynamic replication. Everybody knows how to do it in BSM; but how many know how to do it with jumps, and stochastic volatility, and jumps in volatility?

Propositions abound for alternative models to BSM where the only suggestion for pricing options under jumps and stochastic volatility is simply to take discounted expectations under the risk-neutral probability. (Read the opening of any quant paper.) Yet you see no mention made of dynamic replication. So perhaps I may dare the following question: What if the real reason why Haug and Taleb (and now Triana) are unable to see any future to BSM except in its past, and any model superior to it other than those "developed long before BSM," was that dynamic replication outside the BSM framework is beyond their reach? What if we acknowledged that dynamic replication is indeed the limit of BSM, but decided to *cross* that limit instead of bouncing back to the primitive stage that prohibits replication? What if we resolved that, in order to get the technological process going and deliver a decent derivative pricing framework to the customer, the *first requirement* should be that the model/framework/software/technology first produces the dynamic replication strategy of the derivative?

Traders are dynamic. The market is dynamic. A good trader is his own risk manager; and risk-management is dynamic. To be-in-the-market is to be dynamic. Every option trader of the pre-BSM era must have been a dynamic hedger. (So I fully agree with Haug when, as further argument against the originality of BSM, he claims that dynamic hedging was also available before 1973.) Every option trader worthy of that name must have been dynamically hedging his option trades, without knowing this was called "dynamic replication." How could he not dynamically hedge when he knew the premium he was long or short would vary with the underlying? Even traders who used to hedge options with other options (as Haug insists they did) were *in fact* doing so dynamically.

Except for put-call parity trades, I don't see how traders offsetting options with other options could afford not to rebalance the position every now and then. We are told they were "intuitively" offsetting gamma and vega and not just exposure to the underlying. We are told they were more knowledgeable than post-BSM traders who believed BSM dynamic replication was the answer to everything. But who said the intuition of gamma and vega replication could not be formalized and turned dynamic too in frameworks more general than BSM? BSM's major insight was to put a name, and an algorithm, on such an intuition, regarding only the delta. And they did so, thanks to Merton, by really expressing the infinitesimal P&L of the dynamic, self-financing, portfolio composed of option and underlying and by differentiating derivative values over stochastic paths. Their only sin was to do it assuming no jumps, in a framework (Brownian motion) where the underlying alone is supposed to hedge away all the risk.

When you approach dynamic replication from an incomplete markets perspective, it can no longer be confused with delta-hedging. Under jumps, the option delta is just the Greek that expresses the sensitivity of the option price with respects to tiny movements of the underlying. But this is no hedge. You need a whole new concept of optimal replication in order to track, i.e., replicate, the derivative payoff as closely as possible given jumps. As a matter of fact, your optimal replication strategy, or optimal hedge, will come out different from your delta.[17] Now surely, pre-BSM traders, like Thorp, were dynamically delta-hedging if they owned formulas with a delta very similar to BSM. But what could they do under jumps? How could they make their dynamic hedge work under jumps without first writing down the self-financing infinitesimal expression of P&L, whose first expression we owe to BSM?

Haug's and Taleb's reply may be that hedging with the underlying under jumps will not be robust, be it optimal or not, and that you will need to hedge with options on top of that. I can't agree more! Optimal dynamic hedging in incomplete markets is model-dependent of course. However, this technology can also produce dynamic hedging ratios against options or other instruments than the underlying. And guess what? If the instruments you are using for hedging are carefully chosen, you might manage to have hedging ratios that don't vary a lot.

To repeat, what I like in "dynamic replication" is the word "dynamic." Markets and market-makers are dynamic. The technology of modern option pricing (following BSM) just inserts us in this dynamics. When I suggest that dynamic replication should be our first lead into option pricing and I insist that it carries over to the framework of incomplete markets and jumps, I don't mean to repeat at the next level the same dogmatism and vulnerability to criticism as with the BSM level. As little as I had believed that Brownian motion was everything do I presently believe that jumps or stochastic volatility are the last word. Dynamic replication is only the initiator of the technological process, as you shall see.

The probability distribution, the volatility smile, the supply and demand situation for options, all these are changing and dynamic; so how can we even begin talking of dynamic replication?

Indeed, the real dynamics is what I call the "dynamics of recalibration." However, there

must be something we are recalibrating to begin with, namely a certain model we have a good handle on. For instance, when we say the "volatility smile is dynamic," this presupposes BSM, because the volatility smile is a notion inherited from BSM implied volatility. Knowing the ins and outs of BSM, as Haug once urged in "Know Your Weapon," is just another word for recalibration. No wonder the Greeks that made all the difference there, and were the substance of his elaboration, were vega and all kinds of higher order derivatives of BSM! BSM as such is blind to such Greeks because it assumes volatility is constant. Therefore, talking of knowing those Greeks, as Haug did, is talking of "meta-BSM," in other words, of recalibration.

To my mind, the big problem of derivative pricing is the problem of recalibration. My whole point is that dynamic replication is what gets the problem going because everything is dynamic anyway, and we first need to pin option value on something (typically we start pinning it on the cost of its replication strategy), before we address the much more important and much more interesting problem of recalibration.

Recalibration means we are only observing present tradable prices of options and more generally of derivatives. In essence, recalibration is forward looking, not backward looking. This is why the question has always been, to my mind: "How to price, and hedge, options when all I am observing are the traded prices of options?" and not: "How to price and hedge options given a certain, unwarranted, assumption about the probability distribution?"

To my mind, the most important trading concept that we owe to BSM is the concept of "implied volatility." The funny thing is that Black, Scholes, and Merton were probably unaware of the future success, perhaps even of the coming to existence, of this concept. Pre-1987 traders weren't aware of it either. It took a market crash to give the market back its rights, and to create the implied volatility smile.[18] Now, of course, pre-1973 traders were trading options at prices that diverged from BSM. But I wouldn't call them implied volatility smile traders, because, like I said, the implied volatility smile concept is posterior to BSM.

Yet you ask: Why are we keeping BSM at all, then? Let us forget about it, and no longer even use the expression "volatility smile." Let us directly deal with option prices, without even a representation in terms of volatility, implied or otherwise. I wish the quant finance community could take that liberating step indeed! But even if they did, I would like to preserve the great idea of dynamic replication.

Dynamic replication could have been invented independently of the BSM assumptions of Gaussian distribution, continuous frictionless trading and the like! The significant technological sequence, as I have argued above, is dynamic replication → theoretical pricing → observation of empirical option prices → calibration of your preferred model to option prices (and this step only makes sense because you are rooted in the price processes thanks to dynamic replication) → recalibration.

The question has always been, to my mind: "How to price, and hedge, options when all I am observing are the traded prices of options?"

What we are all looking for, then, is not a "next" model, that lives in some different assumption of underlying stochastic process than BSM. We are looking for the model that lives and endures *in the process of recalibration*. The answer doesn't lie in probability, but in a different philosophical plane.

In sum: the market is a place of prices, not of "states of the world" (what world?) and probability. *Only* prices should qualify as states of the market-world, and *all* the prices should qualify as potential states of that world. Recalibration is just saying that derivatives prices, once they are traded, become states of the world as worthy of our attention as states of the underlying. The problem is, no probability theory and no stochastic process can accommodate this requirement, which is nothing but the reality of the market.

Is there an optimal choice of delta which depends only on observable asset prices?

This "outstanding research question," as Jim Gatheral calls it, concludes a critical study of dynamic hedging where he empirically shows that "delta-hedging is so uncertain that we must delta-hedge as little as possible and what delta-hedging we do must be optimised."[19]

The delta is uncertain because, outside BSM, we need a smile model to explain the vanilla prices and the delta depends on the model. Smile models can agree on the instant snapshot of prices of vanillas (i.e., the implied volatility surface) and fully disagree on its dynamics, i.e., the deltas of the vanillas.

The technology, and the understanding of the problem, has evolved since Gatheral's study. Today, we can find reference points, in the spot market, that inform us about the smile dynamics, therefore help us discriminate between different choices of delta for the vanilla.

These observables are the prices of exotic options. There is correlation, indeed, between the price of a one-touch option (or a forward starting option) and the delta of the vanilla.

So the irony is that we ultimately need to calibrate our model against the prices of exotics in order to infer the right delta for the vanilla! In other words, knowledgeable (or experienced) vanilla traders, as Haug and Taleb like to call them, have now to first become expert exotic option traders! Probably the reason why we'd rather back up and stick with antique models where put-call parity is the only form of arbitrage. God knows indeed where the dynamic curse can lead us!

Yet I do believe this is the way forward. What has changed (and wasn't available in 1904, or

even in the days of Thorp) is *liquidity in the exotics*. I can't help it if trading the vanillas dynamically is becoming more complex today and requires advanced smile models with calibration to exotics! Who knows? Maybe the exotics will become our future vanillas and the vanillas our future exotics.[20] The vanillas are only vanilla relative to BSM, after all!

But to what depth is it necessary to include static controls before we can successfully hedge dynamically?

I'll try to answer this question, and all questions, at once, without going into any depth but by remaining purely on the surface: on the surface of the market. For the market is, to my mind, a constant and continual return, or reversion, to the surface, away from the depth of possibility and metaphysics. Remember I mentioned Deleuze, the great philosopher of the surface and the fold, the great philosopher of the event. To me, the market is a surface without depth, where prices are the only things that take place. The market is the place of events and innovation. Events that are, by definition, unpredictable: in other words, they create the possibilities *that will have led to them*.

Derivative pricing models are all based on possibility and probability, on postulated states of the world and metaphysics. That is to say, they belong to the depth and they tend to produce questions that give the impression of depth. Questions such as: "Is the model true? Is the price right? Will the hedge perform well?" Questions that prompt *questioning*, such as typical of Taleb, namely: "The assumptions are wrong. The random generator has no finite moments. We don't know if we are betting with the right dice, etc." When, the truth is, there is no such depth in the market. There is no random generator or hidden variables to begin with. Why? Because of the *continual resurfacing of the price*; because of the eternal return to the surface that is typical of the market and that is the market.

Indeed, if you really believe in pricing (with models and probability and possibility, I mean) then prices you produce are prices you must trade (otherwise why use models?): prices that become states of the world in their turn, and as such, join the "floor level" of the underlying prices that were the foundation of your initial underlying stochastic process. They just join the floor, the market. If you accept that the market is a place of prices and nothing but prices, hence, that states of the world should only be prices, all the prices and nothing but prices, then you will soon have to admit that the "depth view" (the view of underlying and derivative, of possibility, probability, and pricing of derivative as expectation, etc.) leads to its own ruin.

Every derivative pricing model (or dynamics based on possibility, that is to say based on the "depth") has as consequence that some derivative will ultimately be redundant because you can ultimately perfectly replicate it with a dynamic (possibly multidimensional) hedge. The problem is that no derivative (that is not statically replicable by others) was meant to be redundant. The reason it was created is that it should be traded in its own market, thus bringing innovation, bringing a new price and a new state of the world to the table.

If the surface is all that goes on (and this means that this is always necessarily so, because this is the market necessarily, and this means this is always *necessarily contingent*, for, on the surface, without the depth of possibility to help us predict something, everything is always radically new and unpredictable), then derivative pricing models can have *inversion* as their only fate and destination. We always use them in reverse. We use BSM to compute implied volatility from the price of the traded option. We use smile models to calibrate them to the surface of implied volatilities, etc.

As I argued before, we use models to imprint a dynamics on the surface, not in the depth (as if we really believed in the model as a true possibility or a true generator). And the dynamics we imprint on the surface is the dynamics of *dynamic replication* (which we need in order to remain afloat, to be able to breathe in the next tick and not lose money as soon as the underlying ticks) and, more importantly, the *dynamics of recalibration* of the model, which is the true dynamics, which turns the trader into a swimmer and allows him to go places on the surface, for otherwise he would soon be submerged and drowned by the next recalibration wave.

Typical of the inversion is the fact that we calibrate the dynamics to the prices of vanillas instead of using the dynamics to price the vanillas. We do that in order to price exotics or to compute the delta of the vanillas. Here, Haug and Taleb would argue that they are better off making money without any dynamics, buying and selling options against the obvious other options. Indeed, what is the guarantee that our dynamic hedge will perform well?

How we make sure of our hedge – and this is, for the second time, typical of the inversion – is by calibrating it against the surface of prices of exotics, "thus hedging the hedge," thus inverting the model at the next level: not the level of the price of the vanilla, but the level of its delta. And now the question of how deep the static controls (i.e., the prices of these exotics we are supposedly calibrating against) must be before we can successfully hedge appears as just the rephrasing of the arch-question of markets and trading: "Will we make money?", except that it is now rephrased at the next level, at the level of the hedge not of the price.

I said my answer will come from the surface, not from the depth. I haven't done any studies, as Gatheral did, showing empirically that our dynamic hedge will have performed better when calibrated to the exotics rather than not. All such back-testing is based, indeed, on the idea that there is a hidden, albeit complex, random generator and all we are testing is whether our model, our hedge, etc., has hit the right depth.

I will answer from the surface because the exotic prices we are calibrating against are *prices*, therefore are dynamic, and not static (they are dynamic by the dynamics of recalibration). To hedge our vanilla, it is not enough to hedge it with the underlying: like I said, the "correctness" of the hedge has itself to be hedged with a dynamic hedge including the exotic. So on and so forth. Not to mention that our smile model, which is supposed to price exotics, is now relying on the given market price of the exotic! Eternal return of the surface.

So what are we doing in the end? How are we making money, if at all?

Again, don't expect an answer from the depth. (As if we had found the model for making money.

As if the question concerning the truth of that model made sense.) *We are using this model because we became market-makers in exotics and vanillas*. This is how the market, or the surface, is always ahead of us. Nobody instructed us to make money, as such. We were told to make markets in exotics, to produce original work on that surface, to write on that surface. So we had to come up with a dynamic model.

The market and the surface are moving ahead, whether we like it or not. How could we "write" exotic prices on the surface using only static hedging? We carry this box with us wherever we go, this box containing the model and the depth and the possibility and the probability and the process, however, when we start writing, we rest it on the surface of the table. Like Barton Fink, we write outside the box, yet the box is lying next to us on the table. And, like him, when asked what's inside the box, we answer: "We don't know." And when asked whether what's inside the box belongs to us, we answer: "We don't know."

The right question is not what's inside the box or to whom this belongs (who owns the model? who makes money? etc.). For the right answer is that work can begin and writing can begin after we are altogether freed, not from the box, but from the whole idea of the content of the box. The content is the wrong question to ask. Writing on the surface, this writing capability, can only be conquered *in the third act*. It is only after *both* possibility (the one time possibility of his future work which he thought could be reserved under his name like a hotel room) *and* the sum total of possibilities (or totalitarianism, or dogma, or necessity, as embodied by Karl Mundt, his potent neighbor, who brings an end to the world) had left the scene and he was left alone at his table with only a box to stare at that Barton Fink could begin to write. And this writing had nothing to do with the question of content.

Only after the singularity of the derivative payoff and the totalizing view of the replication algorithm (which brings an end to the market) are done with and left behind can the market-maker start making the market.

We make money if we manage to regain the surface after putting possibility and necessity inside a box yet carrying the box with us wherever we go. The first two acts (the boxing both of possibility and dogma) are the necessary preliminaries in order to *regain the surface*. To become an inspired and productive writer/trader, you need the box. And then you make money by producing original work on the surface, trying your best not to infer any depth from it as you go (any theory, any model), yet remaining active and original.

Pricing and hedging are never absolute. They are relative to the "stratum" in which your market lives: how relatively liquid the exotics are

Like I said, original work is rewarded in the market. And true original work in the market has nothing to do with possibility or with guessing and predicting. It is a pure writing activity.

The surface of writing/pricing is a strange surface indeed. Every sign, or price, you encounter on the surface offers you the opportunity to reconsider all your previous work and to recalibrate. When you recalibrate, it is as if you were saying that the whole story, the whole process, will have had to be different after all, starting all the way back from the origin. But this is not a problem. Who said chronology, antecedent and subsequent, author and copier, cause and effect, are adequate concepts to apply to the market?

How to possibly calibrate to the exotics when, as market-maker dynamic-hedger of vanillas, we live in a market where exotics may not exist yet, let alone be liquid?

Answer: Assume a certain price structure for these exotics (this can be given by your model, free of arbitrage with the vanillas). Use that as internal anchor to your pricing-hedging of the vanillas (or determinant of your smile dynamics). Then, publish those exotic prices to the market. Become a market-maker of exotics as well!

If the market doesn't agree with your exotic prices, they will hit you, you will move your prices, recalibrate your model, readjust your vanillas hedges, and perhaps even reprice the vanillas. Bottom line: here you go; you now have a liquid market on the exotics!

The philosophical bottom line is that there is no ultimate anchor, no underlying stationary dynamics that you hope your model will discover; there is no anchor but the surface of prices. But the operative idea here is that you expect the prices of exotics to be moving "more slowly" than the prices of vanillas. They act as a *relative anchor*. And if they become so liquid in their turn that you need to recalibrate very often, you move to a model of higher order, where the instant smile incorporates both vanillas and first-generation exotics; and you look for second-generation exotics as anchor for the new smile dynamics...

Pricing and hedging are always relative, never absolute. They are relative to the "stratum" in which your market lives: how relatively liquid the exotics are.

There is a paradox in market-making as I said. You have to craft exotics yet you find that ultimately your model will depend on the market price of exotics. This is not a vicious circle. Risk is never laid off completely, and you will always depend on the dynamics and the model that you assume at your specific level. However, *you are an active market-maker*. That's my point. You manage both to make and take the market. You live in between the layers of this globally moving sea. So perhaps the metaphor of the surface shouldn't be taken literally: better to think of a sea with different current speeds at different depths. However, the image of the surface is useful to argue against the depth of metaphysics and the whole idea of a hidden truth...

Who is the market and how are they coming up with their pricing to hit you with prices that allow you to recalibrate?

Who is the market? What's inside the box? What's outside the box?

From my line of argument, it sounds indeed as if the market is the Big Outside.

But how could this outside exist when we are precisely in the process of making it? How could a market on exotics exist, and what's more, help us calibrate a model intended to hedge and price the vanillas, when we are still in the process of *making a market on the vanillas* and when exotics may not yet exist, even conceptually?

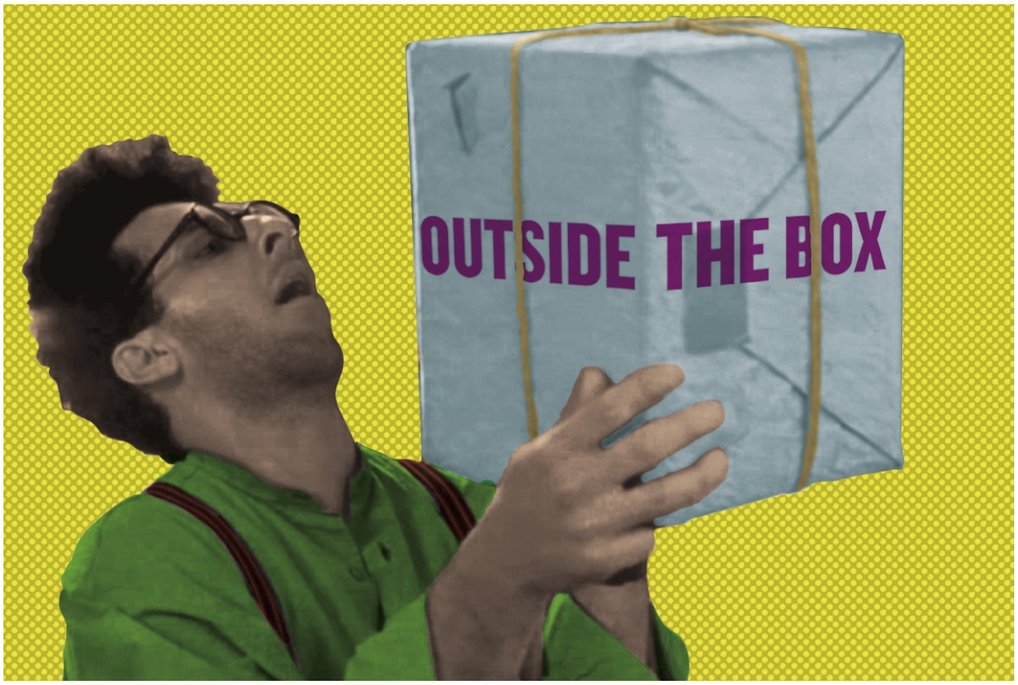
More generally, how could a market exist, when, as quants, all we care about is the model, is the box? Have you noticed that, when quants derive option pricing formulas from stochastic processes they initially assume for the underlying, they do not, at any stage, need the assumption of the existence of a market for the options they are pricing? Not only is the market outside their model, or their scope; it is outside their paper, so to speak. It is outside their philosophical plane, not to mention their job description.

This is the reason they cannot make sense of "implied volatility." What could "implied volatility" be when you look at option pricing from inside the box? What could this really mean that we should invert the pricing formulas against the market prices of derivatives? It means the model is wrong, for sure. Because implying volatility automatically means implying stochastic volatility (as you will basically reimply volatility every day, or recalibrate), and this means implying *volatility smiles*.

So the model is wrong. So someone like Pablo Triana can write: "Black-Scholes became the most popular model because practitioners know that in markets mathematical devices matter most when they can be made to matter least."[21] But why don't we just discard the model, then? If the math has become so irrelevant, why do we stick with the model?

We stick with the model because we use it inverted. We need implied volatility, this false number (false because it relates to a false model), because implied volatility is not a mathematical concept; it is a trading concept.

Certainly, nobody believes that implied volatility is an indication of a truth, lying there, somewhere underneath the surface. It certainly is not the "real volatility" of the underlying, whatever that means. Nor is it the volatility that "the option market is forecasting." The market doesn't know or forecast anything. No, implied volatility is an option-trader concept. It is the first step to answering the big (and I believe, only) question of market-making: "How to price and hedge options when all we are able to observe are option prices?"



The reason I like market-making as the privileged way of making sense of the market is that market-making starts exactly where thought should always start. *It starts in the middle*. If thought's entire ambition and endeavour is to always reach to the outside of thought (for if thought contents itself with the replication of previous thoughts and only of its inside, how can it think new things?) then thought will always have a problem. Indeed, how is thought ever going to even imagine the outside, when this is precisely what lies outside of it? How can thought think outside, or *without*, itself?

The mistake of representational thinking and of the "dogmatic image of thought" (as Deleuze says) has consisted in projecting a *foundation* outside, which was meant to be independent of thought, in order to make the outside stand. To make it stand all by itself, just long enough to allow thought to reach to it in the second instance and claim a new territory, a new discovery. Just so thought could claim that it has achieved its purpose: thinking outside itself. This is how dogmatic thought has invented the categories of truth, or necessity, as the only warrants of the absolute it was reaching for. What's outside? And the answer is: truth, or necessary being. And all that thought has to do is discover that.

This dogmatic projection of the outside is, in fact, only a replica of the inside. It is a fake; it is a representation, in the theatrical sense. Something thought fabricates (hence this cannot be new and cannot be outside) just for the purpose of *staging* its movement to the outside.

To this dogmatic image, which thrives on the metaphor of inside and outside and a wall separating them, which is but the tenacious belief in the idea of *content* (either of the box or of its outside), Deleuze prefers the surface. There is no depth on which to found the surface. You always start in the middle of the surface. However, the surface is not flat. We are not talking of flat

empiricism. The surface is folded and twisted. Deleuze compares it to Möbius strip. You go from the inside to the outside without ever encountering a wall.

What can make thought think, i.e., reach outside itself, are events and encounters. Because events are the only things you can encounter on the surface. The surface is necessarily the place of contingency. It is the other name of the necessity of contingency (yet not of chaos).[22] The question "How to hedge and price when all I observe are option prices" is a question of the surface. Because it is a question, i.e., a challenge for thought, the answer cannot be: "Anything goes." François Zourabichvili (a great commentator of Deleuze) writes: "Deleuze has unceasingly challenged a false alternative, the one that imposes on us to choose between transcendence and chaos, between necessity understood as preexisting truth and the absence of necessity pure and simple."[23]

There is a middle way. Market-makers are thinkers and creators. "Truth is only what thought creates... Thought is creation, not a will to truth..." writes Deleuze (after Nietzsche). Market-makers need both the model and the market. Because they make markets, they need to produce prices as outputs of pricing models. However, because the market is the outside (and cannot be their fabrication), because the market is the outside that they should, as market-makers-thinkers, always be reaching for, they also need prices to be the inputs of their models. I used to say that a market-maker makes a price only insofar as the market makes it.

How to price and hedge on the surface of prices (when all you observe are prices) is the question of the meaning of implied volatility. It is the question that the *derivative pricing technology* (not math) is supposed to address. I tried to work it out in a previous publication[24] and it led me to formulating three rules that every derivative pricing technology must verify: 1) Calibration is for hedging 2) Calibration is recalibration 3) Hedging is for recalibration.

So, to answer the question: "Who is the market and how are they coming up with their pricing to hit me with exotic prices that allow me to recalibrate?", I will first say that this is, at first, internal to my model. I first discover, internally to my model, that different price structures of exotics (say, one-touch options or cliquets) would imply different hedging strategies for the vanillas, if they prevailed.

So how do I know which to choose and which vanilla hedge to apply?

I don't know. I just *think. I take it outside*. I take the inside to the outside. *I publish those prices*.

This is utter nonsense, I grant you, from the quant's point of view. You cannot tell the editor of the mathematical journal, where you intend to *publish your quant paper*, that in order to make sure you got the right dynamics all you have to do is publish exotic prices.

You publish those exotic prices. You write them. This has nothing to do with guessing, or predicting, or thinking that exotic prices are necessarily so. This is just writing. Writing (writing contracts, signing them, etc.) is the first step to the exchange.

You make the market. You prepare yourself to meet somebody out there. At this stage, the answer to "Who is the market?" cannot be representational. It cannot be: "This is the market. This person is the market." These answers are as "full of... content" as the one that says that what's inside the box is true. The right answer should be: "The market is the inside made outside." (This is how what was at first internal to the model becomes external.)

So, is it sufficient to publish those prices to warrant the correctness of the dynamic hedge? To somehow create the truth?

Yes, in a sense. And the strange thing is that, since nobody is trading those exotic prices against you yet and there is no market yet, you cannot use those prices in order to "hedge your hedge." Until somebody hits you and creates the outside for your thought, your hedge is the right one. It is the right one just by dint of your publishing those exotic prices.

Market-making is the activity of the surface. As such it is always transient. In a way, recalibration (as concept, or better, as reality) comes before calibration. The market is not an actuality ("This is the market"), or a possibility ("Let us project what the market is"). It is a *virtuality* (in the Deleuzian sense).

ENDNOTES

1. See Nassim Nicholas Taleb, *Fooled by Randomness*. New York: Random House, 2005.
2. See Donald Mackenzie, *An Engine, Not a Camera: How Financial Models Shape Markets*. Cambridge, MA: MIT Press, 2006.
3. Cf. Haug, E. G. and N. N. Taleb, Why We Have Never Used the Black–Scholes–Merton Option Pricing Formula. *Wilmott*, January 2008, pp. 72–79.
4. Cf. Triana, P., Do Traders Use Black–Scholes? Breakingviews, London, November 2007, www.breakingviews.com.
5. See Nassim Nicholas Taleb, *Dynamic Hedging*. New York: Wiley, 1997.
6. See Gilles Deleuze, *Difference and Repetition,* Paul Patton (trans.). New York: Columbia University Press, 1995.
7. Cf. Gangahar, A., Mispriced Risk Tests Market Faith in a Prized Formula. *Financial Times*, April 16, 2008.
8. Cf. Derman, E., and N. N. Taleb (2005). The Illusion of Dynamic Delta Replication, *Quantitative Finance*, 5(4), 323–326.
9. Or can it? Or rather: do Haug and Taleb perceive how possibly it could?
10. Cf. Haug, E., Know Your Weapon. *Wilmott*, May 2003.
11. Cf. Triana, P., Derivatives Tool that Renders Maths Irrelevant. *Financial Times*, January 15. 2007.
12. Cf. Triana, P., Do Traders Use Black–Scholes? *op. cit*.
13. Cf. Triana, P., Do Traders Use Black–Scholes? *op. cit*.
14. Cf. Ayache, E., The Next Question Concerning Technology: Part II: A World Inverted. *Wilmott*, May 2007, pp. 42–48.
15. Cf. Ayache, E., What Is Implied by Implied Volatility? *Wilmott*, January 2006, pp. 28–35.
16. Cf. Ayache, E., Definitive Smile Model: Part II. *Wilmott*, May 2004, pp. 12-17.
17. Cf. Ayache, E., P. Henrotte, S. Nassar and X. Wang, Can Anyone Solve the Smile Problem?, Wilmott, January 2004: 78-96.
18. Cf. Ayache, E., The Next Question Concerning Technology: Part II: A World Inverted. *op. cit*.
19. Cf. Gatheral, J., Volatility and Hedging Errors. PowerPoint presentation, September 25, 1999.
20. Cf. Henrotte, P., How Exotic Is the Variance Swap? *Wilmott*, November 2006, pp. 24–26.
21. Cf. Triana, P., Derivatives Tool that Renders Maths Irrelevant., *op. cit*
22. See Quentin Meillassoux, *Après la Finitude: Essai sur la Nécessité de la Contingence*. Paris: Editions du Seuil, 2006. English translation, *After Finitude*. (trans. R. Brassier). London: Continuum, 2008 (forthcoming).
23. See François Zourabichvili, *Deleuze: Une philosophie de l'événement*, Paris, PUF, collection Philosophies, 1994, second edition 1997.
24. Cf. Ayache, E., What Is Implied by Implied Volatility?. *op. cit*.