Which model for equity derivatives?

Local volatility was, for a long time, seen as being a universal panacea. However, cracks appeared and we have been forced to look elsewhere for a new framework. Philippe Henrotte, co-founder, partner and head of financial theory and research at Ito33, explores the alternatives

In the equity derivatives space, local volatility has been viewed for a long time as being the final and universal answer to the 'smile problem'. Local academics and practitioners loved this elegant generalisation of the Black-Scholes setting, which is easy to implement on a modified binomial tree and fits any volatility surface. Someone with basic programming skills could quickly derive a pricing and hedging solution for a set of derivatives instruments written on the same underlying security, which would respect the fundamental requirement of absence of arbitrage.

At one point, it seemed all that was needed to price an exotic instrument, no matter how complex, was the input data from a volatility surface. Countless articles would explain how to derive the price of an exotic instrument, such as a barrier option, a forward-starting option or a variance swap, from the knowledge of the volatility surface. Calibration was limited to the derivation of a well-behaved local volatility surface; a numerically ill-posed but computationally straightforward and fast process. Most importantly, this generalisation of the Black-Scholes paradigm vindicated the ubiquitous practice of delta hedging, and any derivative could be exactly replicated by dynamical trades in the underlying. Risk managers were left to worry about vega risk, generalised from a single number under the lognormal diffusion to an array of values in the local volatility framework, a practice called bucketing.

Philippe Henrotte

However, numerous cracks appeared in the local volatility paradigm over the years, which led to the gradual demise of this convenient framework. First and foremost in the equity space, local volatility does not account for the possibility of default. The proponents of the diffusion framework proposed structural models where default was gradually and rationally anticipated by the market, implying no jump and no surprise at the time of default. This generated great academic interest but had little practical sense since default is usually associated with a near collapse of the share price in a sudden and dramatic way. The theoretical prices of the deep out-of-the-money puts are absurdly cheap in any diffusive framework, but fall nicely in line as soon as a jump to default is introduced. An out-of-the-money put with 50% moneyness and one month to maturity is valued at USD 0.0000000007 under Black-Scholes, with 40% volatility and an underlying price normalised at USD 100. Add a jump to default with 1% chance of occurring within a year and the put suddenly costs USD 0.04.

Hedge funds were among the first to spot that the credit default swap (CDS) and the out-of-the-money puts have much in common, and the practice of arbitraging the equity and credit markets gave rise to the equity-to-credit paradigm. It is now well recognised that credit is a major driver of the shape of the smile for single names as soon as their CDS spreads rise above, for example, 100 basis points.

If credit is an inescapable issue for single names, surely it is not relevant in the case of equity indexes. Since most exotic derivatives in the equity space are written on indexes, there is hope in saving the diffusion framework and its ideal tractability when default can be ignored. This reasoning underlies the most recent research efforts in equity derivatives, all dedicated to tackling a second major crack in the local volatility edifice: the production of reasonable prices for exotic instruments.

With no degree of freedom left once the local volatility is derived from the vanilla option prices, it is impossible to fit the prices of additional exotic instruments that do not agree with the local volatility model in the first place. However, it very quickly became apparent to exotic equity desks that the local volatility model mispriced most exotic instruments. The view that this mispricing was only temporary and could be arbitraged away soon became untenable. Quants were under pressure to produce models with additional degrees of freedom that would match the market prices of a few key exotic instruments while still fitting the entire surface of vanilla options. They combined a local volatility parameter to a stochastic volatility model with the hope of getting the best of both worlds: the stochastic volatility would be relevant to the exotic instruments while the local volatility would help produce the desired vanilla option prices.

This approach proved successful for instruments that heavily depend on stochastic volatility. It is certainly clear, for instance, that an option on the VIX will not survive long in a local volatility environment and, symmetrically, a few VIX option quotes will help pinpoint the volatility of the volatility, a parameter that is very difficult to fix from vanilla option prices alone. For long-dated barrier options or forward-starting options, the correlation between the volatility parameter and the price of the underlying is critical. Mixing local and stochastic volatility produces far more realistic dynamics for future smiles than was possible with local volatility alone, and exotic instruments depend as much on the behaviour of future smiles than on the shape of the current one. The cardinal sin of the local volatility setting was to assume that the current smile would determine the process of future smiles.

So much for the good news. The bad news is there is considerably more going on in the equity derivatives space than can be grasped by a mixture of local and stochastic volatility. There is ample evidence that jumps add critical features to the dynamics of the underlying that cannot be tackled under a smooth diffusion setting. Short-dated options are all about jumps and, for the popular weekly options, even small jumps can make a huge difference. Rare, but catastrophic, events cannot be ignored either. They can be described as very large jumps, not only on the underlying price, but also on the volatility, as anyone following the VIX can testify. These extraordinary events are some of the key factors driving the negative skew on the smile and the positive skew on the VIX smile.

Jumps are rather painful to calibrate but are not difficult to model. If jumps are indeed required, why not simply add a few Poisson processes to our preferred diffusion? We would then surely get the best possible model – the Swiss army knife of the equity derivatives quant. Unfortunately, you would still fall short of many critical components. The structure of the jumps is itself stochastic, and so is the correlation between the underlying price and the volatility. Add to the mix dividends and a credit component in the case of single names, both stochastic with a complex correlation structure with the volatility and the jumps, and you get a very complex picture indeed.

All of these features can be nicely embedded in a relatively simple regime-switching model where each regime is a jump-diffusion process with its own level of volatility, jump structure, default intensity and dividend yield. Regimes are selected by a continuous Markov process, and the much-needed correlation between the regimes and the underlying price is achieved by letting the underlying jump during a regime switch. This regime-switching model provides an incredibly rich framework that can morph and adapt itself to a very large number of situations relevant to the equity space.

Think about a company planning a leveraged buyout or a merger. The success or failure of this transaction could mean, respectively, higher or lower volatility, credit rating and dividend yield. And it could be associated with a positive or negative jump on the underlying price. Rather than imposing a rigid scenario to this event, a calibration of the regime-switching model to available derivatives prices will extract from forward-looking instruments the way the market sees this event unfolding.

Calibrating such a rich regime-switching model is not an easy task, particularly since closed-form solutions are typically not available. This probably explains why such simple models have never become mainstream. Research is dedicated to finding models with quasi closed-form solutions, since it is believed that only closed-form solutions can be calibrated in a timely fashion. What matters, however, is not so much the availability of closed-form solutions, it is the ability to solve the direct pricing problem very efficiently. In a regime-switching model with three regimes, for instance, this means solving three coupled partial differential equations or three binomial trees where each node on one tree is possibly connected with the nodes on the other two trees. The gain in terms of modelling versatility is enormous; the pain in terms of computer load is minimal.

In conclusion, I strongly believe that both time and spot homogeneity are essential features of the regime-switching model, and indeed of any decent model in finance. If you need to introduce spot or time dependency in your parameters to fit some market feature, it is most probably a sign that you are missing some stochastic underlying story. It is a shame, for instance, to describe an upward-sloping term structure of CDS spreads with a time-dependent default intensity when two levels of credit and a Markov transition would nicely capture a simple stochastic credit event. The key words here are 'efficiency' and 'robustness'. Ask Isaac Newton what he would think of a model where the field of gravity is tweaked as a function of space and time just to ensure the apple falls squarely on his head.

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Contact
E: info@ito33.com
www.ito33.com